Single-supplier/multiple-buyer supply chain coordination: Incorporating buyers’ expectations under vertical information sharing

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Abstract

We address the coordination problem in a single-supplier/multiple-buyer supply chain. The supplier wishes to coordinate the supply chain by offering quantity discounts. To obtain their complete cost information, the supplier exchanges his own cost parameters with buyers leading to vertical information sharing. The supplier thinks that the buyers, as they have access to supplier’s setup and holding cost information, may demand a portion of the anticipated coordination savings based on the partial information they hold about the cost structure of the entire supply chain. We model each buyer’s expectations based on her limited view of the entire supply chain which consists of herself and the supplier only. These expectations are then incorporated into the modeling of the supply chain, which results in a generalization of the traditional Stackelberg type models. We discuss alternative efficiency sharing mechanisms, and propose methods to design the associated discount schemes that take buyers’ expectations into account. In designing the discount schemes, we consider both price discriminatory and non-price discriminatory approaches. The study adds to the existing body of work by incorporating buyers’ expectations into a constrained Stackelberg structure, and by achieving coordination without forcing buyers to explicitly comply with the supplier’s replenishment period in choosing their order quantities. The numerical analysis of the coordination efficiency and allocation of the net savings of the proposed discount schemes shows that the supplier is still able to coordinate the supply chain with high efficiency levels, and retain a significant portion of the net savings.

Keywords: Supply chain coordination; Inventory management; Discount schemes

1. Introduction

We consider the coordination problem in a single-supplier/multiple-buyer supply chain with decentralized decision making. The buyers face constant and deterministic demand rates, and being separate cost minimizing entities, operate under the assumptions of the traditional Economic Order Quantity (EOQ) model. The supplier is in search of channel coordination policies that will improve his profits. Although the supplier
can infer buyers’ individual demand rates (from buyers’ order quantities and delivery frequency requirements) and, once the demand rates are known, the ratio of the setup and holding costs (from buyers’ order quantities), this information is not adequate to solve the supplier’s profit maximization problem.

Despite this issue, the single supplier/multiple buyers problem has been mostly analyzed as a Stackelberg game where the supplier has access to complete information and coordinates the supply chain as the leader (Chen et al., 2001; Wang, 2001, 2002, 2004). The supplier is entitled to collect a major portion of the net savings of coordination, and it is traditionally assumed that a buyer would accept the solution imposed by the supplier, as long as her cost does not increase, without demanding further benefits. In fact, Wang (2004) notes that this optimistic scenario has its limitations, and there might be cases where buyers require a larger benefit to cooperate with coordination efforts.

In this study, we assume that the supplier engages in vertical information sharing with his buyers with the goal of coordinating the supply chain. By vertical information sharing, we imply that the upstream (i.e., supplier) and downstream (i.e., buyers) participants of the supply chain share information on a 1-on-1 basis, and the supplier does not share a buyer’s private information with others. This leads to the case where the supplier has access to the complete information set that is required to coordinate the supply chain, and, although each buyer has access to the supplier’s setup and holding cost information, individual buyers do not have access to the supplier’s information set that contains information on other buyers. We also assume that the supplier and the buyers will be truthful in information sharing, because of the long-term nature of the relationship they engage in.

Under the vertical information sharing assumption, we model an instance of the supply chain coordination problem in which the supplier does not believe that a traditional Stackelberg type of solution that covers only their additional costs would be willingly accepted by the buyers. Because she now has a limited view of the entire supply chain, each buyer is able to compute some possible coordination benefits based on the information she holds. Indeed, a buyer has complete information regarding the supply sub-chain that consists of herself and the supplier. The supplier therefore models each buyer as an entity with possibly positive expectations based on the supply sub-chain, and formulates his coordination problem with the objective of maximizing his own returns while guaranteeing that each buyer obtains a share from the coordination benefits that satisfies her expectations. We formulate the problem as a constrained Stackelberg game where the supplier is still attributed the leader role, and the supplier believes that a buyer will cooperate only if her return is in line with her expectations she has developed based on the partial information she holds. To the best of our knowledge, this paper presents the first attempt at incorporating buyers’ anticipated net savings into the supply chain coordination problem.

The multi-echelon inventory literature has dealt with the single supplier/multiple buyers coordination problem under a centralized decision making, or cooperative structure. It has been shown that minimizing system cost in a centralized system may require non-stationary replenishment intervals (Graves and Schwarz, 1977). A stationary policy calls for constant replenishment periods that repeat themselves over time, whereas a non-stationary policy involves changing the replenishment periods over time. Non-stationary replenishment intervals have been deemed difficult to implement in real settings (Wang, 2001). The integer-ratio and power-of-two policies developed by Roundy (1985) have become landmark approaches to the centralized coordination problem. An integer ratio policy requires the ratio of the buyers’ replenishment intervals over the supplier’s replenishment interval – or the reciprocal of this quantity – to be an integer. Likewise, a power-of-two policy requires this integer to be equal to some power of two. These practically implementable policies have proven error bounds over all possible policies, including non-stationary ones, that are quite tight.

In a decentralized decision making setting, supply chain coordination is to be achieved via some form of sharing of coordination benefits among channel members. Traditionally, the supplier designs a quantity discount scheme, and when buyers choose their new order quantities under the quantity discount scheme offered by the supplier, the coordinated solution is achieved. The pivotal point in the design process of the quantity discount scheme is the supplier’s ability to model the buyers’ reactions to the quantity discount scheme. In one of the earlier works on this problem, Lal and Staelin (1984) develop an incremental quantity discount scheme with a unified pricing policy. A unified pricing policy is basically a discount scheme that does not discriminate among the buyers. In the United States, this policy is in line with the Robinson–Patman Act (RP Act) which specifies that it is unlawful to discriminate among buyers by offering them different prices for the same or similar commodities. Wang (2002) addresses the supply chain coordination problem using the cost structure of
Lal and Staelin (1984), and offers a unified and discrete all-units quantity discount scheme. In dealing with heterogeneous buyers, however, the supplier is assumed to incur setup costs specific to each buyer’s individual orders. This assumption implies that there is no cost savings potential in coordinating the timing of the orders of different buyers. In a subsequent paper, Wang (2001) incorporates an additional fixed setup cost for the supplier for each order into the overall cost structure and makes time coordination potentially beneficial. Due to this new setup cost and modified holding cost structure for the supplier, the lot sizing decisions become part of the problem. The coordination is then achieved by designing a unified all-units quantity discount scheme and a power-of-two type of policy that maximize the supplier’s annual profit. Wang (2004) addresses the same problem, and presents an integer-ratio policy. A common feature of the discount schemes presented in Wang (2001, 2004) is that the buyers are required to comply with the base planning period which is set by the supplier. Because of this additional requirement, the proposed quantity discount schemes do not fully comply with the RP Act. In Section 2.2.4, we provide a detailed numerical example that illustrates how time coordination is achieved through this additional requirement in Wang (2001).

In other related work, Chen et al. (2001) show that a discount based on quantities only may not be sufficient to achieve channel coordination in a setting where each buyer’s demand is assumed to be a decreasing function of the retail price. They argue that the same level of total benefits as in the centralized system can be achieved via periodically charged fixed fees and a nontraditional discount scheme that is based on annual sales volume, order quantity and order frequency. As it is argued in Wang (2001), this may be in contradiction with the RP Act in the United States, and such solutions may not be legally implemented. The RP Act has been enacted to protect competition and has been heavily criticized for not serving its purpose and causing economical inefficiencies in markets. This discussion has made its way into economics textbooks, and examples that demonstrate efficiency losses due to the inability of offering different prices to the buyers of the same commodity are presented, for instance, in Viscusi et al. (2000). It is also mentioned in Tirole (1988) that the Act has not been strictly enforced. Yet the RP Act’s enforcement is open to private litigation, and therefore ignoring it may have serious consequences for businesses.

In this paper, we consider the single-supplier/multiple-buyer supply chain coordination problem where demand faced by each buyer is not price-sensitive. Each buyer is an independent decision making entity whose objective is to minimize her total annual cost that consists of setup and holding cost components under the traditional assumptions of the EOQ model. We model the supplier as an entity with the objective of minimizing his total cost that consists of setup and holding cost components, and assume that he incurs a setup cost for every shipment or production batch that may cover one or more buyers’ orders. Therefore, prior to any coordination effort, the supplier’s problem is an infinite horizon lot-sizing problem where each buyer’s orders arrive with a fixed frequency. In order to form a benchmark for the evaluation of the coordination benefits, we model the supplier’s problem in an approximate manner by discretizing the time line using a unit period of time, such as a day or a week, and assuming that the supplier accepts orders that are compatible with this restriction. Note that, when the unit time period is sufficiently small such as one day, this provides a reasonable approximation of the continuous infinite horizon problem. The resulting problem can be formulated as a Mixed Integer Programming (MIP) problem in the spirit of the work of Kalymon (1972). A similar benchmark is used in Wang (2001) to measure the effectiveness of coordination strategies.

We formulate the supply chain coordination problem as a constrained Stackelberg game where the supplier’s objective is to maximize his own benefits while meeting possibly positive benefit expectations of the buyers. More precisely, the amounts incorporated as constraints are the supplier’s estimates of the buyers’ expectations. To solve this problem, we restrict our attention to integer ratio policies of Roundy (1985) again on a discretized time line. Moreover, we incorporate the problem of designing an all-units quantity discount scheme into the coordination problem. An important feature of our discount schedule is that it does not rely on a base planning period (see Wang, 2001, 2004) to achieve time coordination. In other words, the supplier does not explicitly require buyers to comply with his base planning period, i.e., his replenishment interval, to achieve time coordination. The discrete time line allows us to formulate an MIP that incorporates the buyers’ expectations as constraints. The optimal solution of the MIP gives us a discount schedule that achieves supply chain coordination while meeting the expectations of buyers. The discrete time line assumption is implicit in our work, and when we refer to an “optimal” solution, for instance, we mean optimality with the discretized time line assumption. We would like to remark that this is an approximation of the continuous version of the
problem, and through our numerical analysis we try to demonstrate that the approach constitutes a quite reasonable approximation.

Due to vertical information sharing, the supplier has full information about the entire supply chain, and each buyer has partial information – that of her own and the supplier’s – about the supply chain parameters. We introduce the notion of a supply sub-chain that consists of the supplier and one buyer to be able to model the viewpoint of a buyer with full information. The buyer possesses full information regarding her own supply sub-chain and is able to compute the coordination benefits of a proposed order quantity for this particular sub-chain. Therefore the supplier believes that a buyer would comply with a discount schedule that leads to a benefit matching this estimate. More specifically, we model each buyer as anticipating at least half of the gain associated with a new order quantity. In other words, we assume that the supplier is willing to share the coordination benefits equally, where the benefits are computed according to the supply sub-chain. The equal sharing of the virtual benefits has the property of seeming to attribute equal power to both parties and indeed corresponds to a Nash Equilibrium in a two-party bargaining environment where both parties are risk neutral as discussed in Kohli and Park (1989). From a technical point of view, this is a simple parameter for our model and can be easily modified.

We propose a variation of the simple supply sub-chain model in which supplier’s total demand information is available to individual buyers. Although the supplier is not required to share his total demand information with buyers, in certain cases the buyers may be able to infer this quantity, for instance, based on the industry records. This variation helps us numerically analyze the impact of this piece of information, and higher expectation estimates, on the supply chain coordination problem.

The modeling environment of the paper serves two main purposes. First, buyers’ expectations are incorporated into the supply chain coordination problem through vertical information sharing. Second, the modeling approach does not rely on buyers’ compliance with the base planning period of the supplier to achieve time coordination. Another important aspect of the paper is the operationalization of the supply chain coordination models under the discrete time line assumption. The discretized time line assumption is particularly needed for solving the supplier’s lot sizing problem prior to coordination. Our use of Kalymon (1972)’s work to model the supplier’s lot sizing problem prior to coordination is discussed in Section 2.1, and documented in detail in Appendix A. Modeling of buyers’ expectations and the discount scheme design problem are presented in Section 2.2. The discount scheme design problem is solved by utilizing a line search procedure on the supplier’s replenishment period. For a fixed value of the supplier’s replenishment period, an MIP problem is solved to minimize the supplier’s total cost, which also includes the total discount offered to the buyers. The line search procedure uses a lower bound on the total cost of the supplier to reduce the search space. The discount scheme design problem is solved in both the presence and the absence of the non-price discrimination requirement imposed by the RP Act. A study of numerous experiment factors is also reported. Section 4 contains our concluding remarks.

2. The model

We assume that a supplier distributes a single product to $N$ buyers, and no shortages are allowed in the distribution system. In line with the assumptions adopted in Wang (2001), we assume that there exists an initial wholesale price that has been established by the supplier, and each buyer determines her own annual demand, and the order quantity and frequency, i.e., EOQ, under this wholesale price. We also assume that each buyer’s demand is not price-sensitive, i.e., the supplier cannot change a buyer’s annual demand by changing the wholesale price of the product. The motivating factors behind this assumption, along with the restrictions imposed by it, are discussed in Wang (2001). The supplier replenishes his inventory through a manufacturing facility with an infinitely large production rate, or with instantaneous deliveries from a source with no capacity restrictions. Also in parallel with the literature, the assumption that the buyers’ effective holding cost is independent of the quantity discounts will be made in our subsequent analysis. Munson and Rosenblatt (2001) analyze the impact of this assumption on the cost structure of the problem, and report that average increase in the total cost of the buyer is less than 0.022% over a set of 500,000 problems.
We start the description of the modeling environment with the following notation:

- \( N \) number of buyers,
- \( S \) supplier’s fixed setup cost for every order he places,
- \( v \) variable cost of the product for the supplier, to be used to compute supplier’s holding cost,
- \( S_i \) buyer \( i \)'s fixed setup cost for every order she places with the supplier, \( i = 1, 2, \ldots, N \),
- \( v_i \) variable cost of the product for buyer \( i \), to be used to compute buyer \( i \)'s holding cost,
- \( D_i \) buyer \( i \)'s annual demand,
- \( D \) total demand of the supply chain, where \( D = \sum_{i=1}^{N} D_i \),
- \( N \) number of buyers,
- \( r \) inventory holding cost rate per year (interest rate),
- \( Q_i^0 \) buyer \( i \)'s initial EOQ that is compatible with the unit time restriction,
- \( k_i^0 \) buyer \( i \)'s initial replenishment period, i.e., buyer \( i \) places an order every \( k_i^0 \) time units, where \( k_i^0 = \frac{Q_i^0}{D_i/r} \), and integer-valued,
- \( TCS(Q) \) buyer \( i \)'s total setup and holding cost as a function of the order quantity \( Q \).

Under the unit time restriction, the EOQ of buyer \( i \) can be determined using the convexity property of the total cost function \( TCS(Q) \), which is given by \( TCS(Q) = \frac{D_i}{Q} S_i + \frac{Q}{2} r v_i \). The EOQ of buyer \( i \), i.e., \( Q_i^0 \), would be equal to \( k_i^0 \frac{D_i}{r} \), where \( k_i^0 \) is equal to either \( \left\lfloor \frac{\sqrt{2Sn}}{Q} \right\rfloor \) or \( \left\lceil \frac{Q}{2} \sqrt{\frac{Q}{D_i/r}} \right\rceil \), depending on which of these solutions yields a lower total cost. Note that \( \tau \left( \tau \sqrt{\frac{2 Q_i^0}{D_i/r}} \right) \) is the smallest (largest) order quantity that is compatible with the unit time restriction and larger (smaller) than the optimal order quantity of the problem without the unit time restriction, and, since \( TCS(Q) \) is convex in \( Q \), one of these solutions would be the optimal order quantity.

2.1. Supplier’s optimal replenishment policy without coordination

We first address the supplier’s optimal replenishment problem when there exists no coordination in the supply chain. After the buyers select their \( Q_i^0 \) values, i.e., EOQs, the supplier can design a replenishment policy with the objective of minimizing his total cost. The solution of this problem constitutes a basis for evaluating efficiency of various coordinated solutions. We now introduce the following notation:

- \( RP^0 \) supplier’s optimal replenishment policy when buyers’ initial order quantities are \( Q_i^0 \), \( i = 1, 2, \ldots, N \),
- \( TCS(RP) \) supplier’s total cost with a replenishment policy of \( RP \).

We propose to approximately solve this problem with the additional restriction of unit time structure, using an approach introduced in Kalymon (1972) for finite horizon arborescence type of inventory systems. Kalymon (1972)’s model uses a discretized time line, and each buyer’s requirements over the planning horizon can be variable. In our case, buyer \( i \) requires a delivery of \( Q_i^0 \) units every \( k_i^0 \) time units. Since the time horizon of our problem is not finite, we will create renewal points (as, for example, discussed in Graves and Schwarz, 1977) in the time line, and solve the problem for one renewal period to find supplier’s replenishment policy. The resulting lot-sizing problem can be solved using a standard MIP formulation of the problem (see Appendix A).

The solution of the lot-sizing problem would be the supplier’s optimal replenishment policy, \( RP^0 \) for the selected renewal cycle. The sum of the objective function value of this problem, i.e., \( TCS(RP^0) \), and the buyers’ total initial cost, i.e., \( \sum_{i=1}^{N} TCS_i(Q_i^0) \), would constitute a basis for measuring the efficiency of the coordinated solutions. We would like to note one more time that this is an approximate way of solving the original infinite horizon problem, however, since there exists no optimal solution methodology for the original problem that can be implemented practically, \( RP^0 \) can be considered as the best policy of the supplier without coordination.

2.2. Supplier’s optimal replenishment policy with coordination

In this section, we model the supplier’s coordination problem as a constrained Stackelberg game. We will first discuss how each buyer’s expectations are modeled under vertical information sharing, and then
determine the optimal coordinated policy of the supplier as an integer ratio policy that is compatible with the individual expectations of the buyers. Along with the optimal coordinated solution, we will also design a discount scheme that will be used to implement the coordinated solution. According to the RP Act, the discount scheme that the supplier offers to the buyers should be non-price discriminatory, and we will first design a discount scheme that is in accordance with this requirement. We finally discuss the design of the discount scheme by relaxing the non-price discrimination requirement. The discount scheme design problem generates a quantity discount schedule which ensures that each buyer, with the objective of minimizing her own total cost, chooses the intended quantity as her optimal solution.

2.2.1. Buyers’ expectations

As we have argued in the introductory section, due to the vertical information sharing setting of the problem, the supplier believes that the buyers have expectations for possibly positive returns from a coordinated solution. Therefore to ensure their cooperation, the supplier needs to transfer an amount in line with their individual expectations in any solution proposed to them. As a justifiable approach, we quantify these expectations by modeling a supply sub-chain for each buyer that consists of herself and the supplier. Note that the coordination of the sub-chain is discussed with the goal of establishing a basis for the buyers’ expectations and is not intended for implementation. In that sense, the costs and savings discussed in this section are virtual, i.e., they are not necessarily the exact costs or savings to be realized by the parties when the actual coordinated solution is implemented. We will need the following additional notation to express buyers’ expectations:

\[ SC_i \quad \text{sub-chain that consists of buyer } i \text{ and the supplier}, \]

\[ Q_i^c(\cdot) \quad \text{coordinated order quantity for } SC_i \text{ when this sub-chain is coordinated in isolation}, \]

\[ n_i(Q) \quad \text{optimal lot-size multiplier of the supplier in } SC_i \text{ when the order quantity of buyer } i \text{ is } Q, \]

\[ TC_{SC_i}(Q, n) \quad \text{total setup and holding cost of the supplier in } SC_i \text{ when the order quantity of the buyer is } Q \text{ and the lot size multiplier is } n : TC_{SC_i}(Q, n) = S_S \frac{D_i}{n} + (n - 1) \frac{Q}{2} v_i r, \]

\[ Q_i^{\text{IR}} \quad \text{an order quantity the supplier wants buyer } i \text{ to choose in the actual coordinated solution to be implemented}, \]

\[ NS_{SC_i}(Q) \quad \text{net savings of sub-chain } i, \text{ as computed by buyer } i, \text{ when buyer } i \text{ uses an order quantity of } Q, \text{ instead of her initial order quantity of } Q^0, \text{ where } NS_{SC_i}(Q) = TC_{SC_i}(Q^0, n_i(Q^0)) + TC_i(Q^0) - TC_{SC_i}(Q, n_i(Q)) - TC_i(Q). \]

When the supplier announces a discount schedule, buyers are able to observe the actual order quantity that the supplier wishes to impose upon them in the coordinated solution, \( Q_i^{\text{IR}} \), but they are not able to compute the true savings associated with this quantity due to their lack of complete information about the entire supply chain. Instead, they are able to compute a virtual savings quantity based on their individual supply sub-chain for which they possess complete information.

The anticipated net savings of buyer \( i \) is then equal to \( ANS_i(Q_i^{\text{IR}}) \), where, for a given order quantity of \( Q \), \( ANS_i(Q) \) is given by

\[
ANS_i(Q) = \max \left\{ 0, \frac{1}{2} NS_{SC_i}(Q) \right\}.
\]

Note that, when \( Q \) is different than \( Q^0 \), it is possible for an individual sub-chain \( i \) to incur a net loss due to coordination when considered in isolation. In that case, we assume that the anticipated net savings of buyer \( i \) would be zero, i.e., the buyer would not be willing to share any additional virtual cost with the supplier.

We propose a variation of the above estimate to address the case where buyers are able to infer the total demand that the supplier faces, i.e., \( D \). In this case, the supply sub-chain may be remodeled to incorporate this information because each buyer is aware of her share in the entire supply chain. With total demand information, a buyer could estimate the net savings of the entire chain by using her own cost information which is equal to \( \frac{D}{2} NS_{SC_i}(Q^0) \) (Appendix B). This corresponds to assuming that the buyer works with the supplier as his only outlet, and the computed quantity reflects the impact of buyer \( i \)'s cost structure on the coordination efficiency. Buyer \( i \) may now expect a share of these estimated virtual savings in line with her share in the total demand of the supplier, which is given by \( \frac{D}{2} \sqrt{\frac{2}{D} NS_{SC_i}(Q^0)} = \sqrt{\frac{2}{D} NS_{SC_i}(Q^0)} \), provided that this quantity is
larger than $\text{ANS}_i(Q)$ as computed above. In other words, in the known total demand case, buyer $i$’s anticipated net savings function is given by

$$\text{ANS}_i(Q) = \max \left\{ \text{ANS}_i(Q), \frac{1}{2} \sqrt{s} \text{NS}_{\text{SC}}(Q_i) \right\}. \tag{2}$$

Therefore, in the known total demand case, buyer $i$ would expect to receive a net savings of $\text{ANS}_i(Q_{i\text{IR}})$ as a result of the supply chain coordination implementation.

In both of the above cases, we propose an equal sharing pattern to define the portion of the net savings to be transferred to a buyer by the supplier. This corresponds to a Nash Equilibrium in a bargaining environment when both parties are risk neutral (Kohli and Park, 1989). By adjusting the multiplier in the above formulas, it is possible to accommodate alternative sharing patterns among parties. The net savings amount transferred to a buyer corresponds to the level at which the supplier expects her cooperation by subscribing to the quantity discount offered to her, and may be adjusted according to the supplier’s judgment.

2.2.2. Optimal integer ratio policy under the Robinson–Patman Act

In this section we will present a procedure to design an integer ratio policy in conjunction with a discount scheme that would minimize the supplier’s total cost which consists of holding and setup costs, and the discount offered to the buyers to implement the policy. The optimization problem will be modeled as a constrained Stackelberg game, where each buyer’s expectations about her savings in a given solution will be taken into consideration in designing the discount scheme. The discount scheme is also required to be non-price discriminatory in this section. We will relax this requirement in the next section, and subsequently discuss the impact of this requirement on the net savings of the supplier and the efficiency of the supply chain in the numerical analysis section.

A similar discount scheme design problem is introduced and solved in Wang (2001). Wang (2001) develops his non-price discriminatory discount scheme under two assumptions: (1) buyers’ net savings expectations are basically equal to zero, and (2) a larger buyer (i.e., a buyer with a higher demand level) has a larger EOQ and a lower ratio of holding cost to demand. These assumptions make it possible to outline a numerical algorithm instead of solving a nonlinear mixed integer programming problem in Wang (2001). As mentioned before, we incorporate buyers’ net savings expectations as possibly positive values, and therefore the first assumption does not hold in our case. We also relax the second restrictive assumption to cover a broader range of problems. Another feature of our modeling environment is the discretized time line. We are able to deal with this more general problem since we take a mathematical programming approach details of which will be given below.

We will address this problem within the scope of integer ratio policies. We first present an MIP formulation that solves the coordination and discount schedule design problem for a fixed replenishment interval of the supplier. We then present a simple search algorithm over possible replenishment intervals of the supplier to find the overall optimal solution. We now introduce the following parameters and decision variables of the optimization problem:

**Parameters**

- $T_S$: fixed replenishment interval of the supplier, in terms of number of unit time periods,
- $H_{i,k}$: annual holding cost of the supplier regarding buyer $i$’s orders when buyer $i$’s replenishment interval is $k$ unit time periods, $k = 1, 2, \ldots, K$, where

$$H_{i,k} = \begin{cases} \infty & \text{if } \frac{T_S}{k} \text{ or } \frac{k}{T_S} \text{ is not an integer,} \\ 0 & \text{if } T_S \leq k \text{ and } \frac{k}{T_S} \text{ is an integer, and} \\ \left(\frac{T_S}{k} - 1\right) \frac{k}{T_S} + T_S & \text{if } T_S > k \text{ and } \frac{T_S}{k} \text{ is an integer, and} \end{cases}$$

$K$ is sufficiently large. When $T_S > k$, the supplier starts its replenishment interval with $\frac{k}{T_S}T_S$ units of inventory dedicated for buyer $i$’s delivery requirements in the replenishment interval. After the first delivery in the beginning of the replenishment interval, the inventory level becomes $\frac{T_S}{k}(T_S - k)$. From this maximum inventory level, we can compute the average inventory as $(\frac{T_S}{k} - 1) \frac{k}{T_S}$. 


$Q_{i,k}$ buyer $i$'s smallest order quantity that is compatible with the unit time requirement and greater than buyer $j$'s order quantity that corresponds to a replenishment interval of $k$ unit time periods, where $Q_{i,k} = \left[ \frac{Q_i}{\tau} \right] \frac{Q_i}{\tau}$.

$C_{i,k}$ buyer $i$'s total cost with an order quantity of $Q_{i,k}$, where $C_{i,k} = \frac{D_i}{Q_{i,k}} S_i + \frac{Q_{i,k}}{2} v_i r$.

$MD_{i,k}$ anticipated net savings of buyer $i$ when her replenishment interval is $k$ unit time periods, where

$$MD_{i,k} = \max \left\{ 0, \frac{1}{2} NSC_i \left( \frac{D_i}{\tau} k \right) \right\},$$

or

$$MD_{i,k} = \max \left\{ 0, \frac{1}{2} NSC_i \left( \frac{D_i}{\tau} k \right), \frac{1}{2} \sqrt{\frac{D_i}{\tau}} \sqrt{NSC_i (Q_i^p)} \right\},$$

depending on the availability of the total demand information to the buyers.

**Decision variables**

$X_{i,k}$ binary decision variable for buyer $i$'s replenishment interval, where $X_{i,k} = 1$ if the replenishment interval is $k$ unit time periods, 0 otherwise.

$BP_i$ the break-point that corresponds to the quantity the supplier wants to have buyer $i$ choose as her optimal decision when offered the quantity discount scheme.

$DC_i$ per unit all-units quantity discount that is offered if the order quantity is greater than $BP_r$.

The MIP formulation presented below is developed to find a coordination policy and a discount scheme to implement it when the supplier’s replenishment interval is fixed at $T_S$. The objective is to minimize the supplier’s total cost, and the replenishment intervals of buyers, and hence their order quantities, are decision variables that can take on values that are in accordance with integer ratio policies.

$$(DSDP: T_S) \quad TC_S^{DC} (RP: T_S) = \min S_T \quad \frac{\tau}{T_S} + \sum_{i=1}^{N} \left( DC_i D_i + \sum_{k=1}^{K} H_{i,k} X_{i,k} \right)$$

subject to

$$\sum_{k=1}^{K} X_{i,k} = 1, \quad i = 1,2,\ldots,N, \quad (3)$$

$$BP_i = \sum_{k=1}^{K} Q_{i,k} X_{i,k}, \quad i = 1,2,\ldots,N, \quad (4)$$

$$BP_i \geq Q_i^p, \quad i = 1,2,\ldots,N, \quad (5)$$

$$DC_i D_i - \left( \sum_{k=1}^{K} C_{i,k} X_{i,k} - TC_i(Q_i^p) \right) \geq \sum_{k=1}^{K} MD_{i,k} X_{i,k}, \quad i = 1,2,\ldots,N, \quad (6)$$

$$(DC_i \leq DC_j) \quad \text{and} \quad (BP_i \leq BP_j)$$

or

$$(DC_i \geq DC_j) \quad \text{and} \quad (BP_i \geq BP_j), \quad i,j = 1,2,\ldots,N, \quad (7)$$

$$(BP_i \geq Q_i^p)$$

and

$$\left( DC_j D_j - \left( \sum_{k=1}^{K} C_{i,j,k} X_{i,j,k} - TC_i(Q_i^p) \right) \leq DC_i D_i - \left( \sum_{k=1}^{K} C_{i,j,k} X_{i,j,k} - TC_i(Q_i^p) \right) \right)$$

or

$$(BP_j \leq Q_i^p) \quad \text{and} \quad \left( DC_j D_j \leq DC_i D_i - \left( \sum_{k=1}^{K} C_{i,j,k} X_{i,j,k} - TC_i(Q_i^p) \right) \right), \quad i,j = 1,2,\ldots,N, \quad (8)$$

$$X_{i,k} \in \{0,1\}, \quad i = 1,2,\ldots,N; \quad k = 1,2,\ldots,K. \quad (9)$$
Constraint set (3) ensures that a replenishment interval is selected for each buyer. Due to the definition of the \( H_{i,k} \) parameter, the selected interval will comply with the integer ratio policy requirement. Constraint set (4) computes the order quantities that correspond to buyers’ replenishment intervals. These order quantities, which are denoted by \( BP_i = 1, 2, \ldots, N \), will also constitute the break points of the discount scheme that the supplier will present to the buyers. Constraint set (5) ensures that the break point, or designated order quantity, of each buyer is greater than or equal to her initial order quantity. This constraint set is required to make sure that each buyer selects her designated order quantity when she solves the optimization problem to find her optimal order quantity under the quantity discount scheme offered by the supplier. When \( BP_i < Q_i^0 \) is allowed, buyer \( i \)'s original optimal order quantity would be feasible for one of the quantity discounts corresponding to one of the break points that is higher than \( BP_i \), as the discount given at this higher level will be higher than the one to be collected at \( BP_i \). Therefore, it will become impossible for the supplier to impose \( BP_i \) as the new order quantity on buyer \( i \). Constraint set (6) ensures that each buyer’s net savings, i.e., the difference between the discount the buyer obtains at her designated break point and the additional cost she incurs with that order quantity, is greater than or equal to her anticipated net savings. Constraint set (7) ensures that the discount scheme is non-price discriminatory, i.e., the discount is always higher with a higher order quantity. To simplify the exposition, constraint set (7) is expressed in disjunctive format, and can be readily converted into integer-linear constraints with the help of \( N^2 \) additional binary variables. Constraint set (8) ensures that, with the resulting discount scheme, buyer \( i \)'s optimal strategy is to choose \( BP_i \) as her new order quantity. This is achieved by making buyer \( i \)'s net savings at break point \( BP_i \) greater than or equal to her net savings at any other break point. Note that, when a break point \( BP_i \) is smaller than the initial order quantity \( Q_i^0 \) of buyer \( i \), we may assume that the buyer \( i \)'s initial order quantity will be valid for the quantity discount \( DC_j \) that corresponds to break point \( BP_i \), and compute her net savings as \( DC_jD_i \). This assumption will be definitely valid for the largest break point that is smaller than \( Q_i^0 \), and, since the discount scheme offers increasing discounts with increasing order quantities, the other constraints would be automatically redundant. Constraint set (8) too is expressed in disjunctive format to simplify the exposition, and can be readily converted to integer-linear constraints.

The problem (DSDP: \( T_S \)) finds the buyers’ replenishment intervals in conjunction with a discount scheme that minimizes the total cost of the supplier when his replenishment interval is fixed as \( T_S \). Since our time line is discrete, the overall optimal replenishment interval of the supplier can be determined by performing a search over all possible values of \( T_S \), and using a lower bound on \( TC^{DC}(RP : T_S) \). The lower bound can be easily developed by using Roundy (1985)’s lower bound on total cost of the supply chain when the replenishment interval of the supplier is fixed. Let \( TC^{LB}_{SC}(T_S) \) be the lower bound of Roundy (1985), as computed in Lu and Posner (1994), on the total cost of the supply chain when supplier’s replenishment interval of the supplier is fixed as \( T_S \). The objective function of (DSDP: \( T_S \)), i.e., \( TC^{DC}_{SC}(RP : T_S) \), has two components: supplier’s total setup and holding cost and the total discount offered to the buyers. The total discount offered to the buyers is greater than or equal to their incremental cost in the optimal solution (constraint set (6)). Therefore, supplier’s total setup and holding cost plus the total incremental cost of buyers constitutes a lower bound on the objective function value of (DSDP: \( T_S \)). Since the total initial cost of the buyers, i.e., \( \sum_{i=1}^{n} TC_i(Q_i^0) \), is a fixed quantity which is greater than the total initial cost of buyers without the discrete time line assumption, \( TC^{LB}_{SC}(T_S) - \sum_{i=1}^{n} TC_i(Q_i^0) \) constitutes a valid lower bound on \( TC^{DC}_{SC}(RP : T_S) \):

\[
TC^{DC}_{SC}(RP : T_S) = \min_{s.t. (3)-(9)} \left( S_S \frac{\tau}{T_S} + \sum_{i=1}^{N} \left( DC_iD_i + \sum_{k=1}^{K} H_{i,k}X_{i,k} \right) \right)
\geq \min_{s.t. (3)-(9)} \left( S_S \frac{\tau}{T_S} + \sum_{i=1}^{N} (TC_i(BP_i) - TC_i(Q_i^0)) + \sum_{i=1}^{N} \sum_{k=1}^{K} H_{i,k}X_{i,k} \right)
\geq TC^{LB}_{SC}(T_S) - \sum_{i=1}^{N} TC_i(Q_i^0).
\]

By an argument similar to given in Roundy (1985), this lower bound is convex in \( T_S \).
The simple search procedure to find the overall optimal solution for the supplier can be outlined as follows. We first determine the replenishment interval of the supplier that minimizes $TC_{\text{LB}}(\cdot)$ under the continuous time line assumption. Let $T_{\text{LB}}^S$ be the optimal replenishment interval that minimizes $TC_{\text{LB}}(\cdot)$. $T_S$ is initialized as 1, the mixed integer program (DSDP: $T_S$) is solved, and the upper bound $UB$ is set equal to the objective function of (DSDP: $T_S$) with $T_S = 1$. The problem (DSDP: $T_S$) is then solved for $T_S = 2, 3, \ldots, \lfloor T_{\text{LB}}^S \rfloor$, and the upper bound $UB$ is updated if an improved $TC_{\text{DC}}(\cdot)$ value is obtained. The problem (DSDP: $T_S$) is now solved for $T_S$ values larger than $\lfloor T_{\text{LB}}^S \rfloor$, however, because of the convexity property of $TC_{\text{LB}}(T_S)$, the search is stopped when $TC_{\text{LB}}(T_S) - \sum_{i=1}^{n} TC_i(Q_i^0)$ is greater than the upper bound $UB$, with $UB$ being the optimal solution of the problem.

2.2.3. Optimal integer ratio policy without the Robinson–Patman Act

When the non-price discrimination requirement is relaxed, the optimal discount scheme can be determined by solving (DSDP: $T_S$) after constraints which guarantee that the discount scheme is non-price discriminatory, i.e., constraint sets (5), (7), and (8), are removed from the formulation. This makes it possible for the supplier to offer customized discounts for each buyer that are not necessarily a non-decreasing function of order quantity.

2.2.4. Comparison of quantity discount schemes

In this section, we present a numerical comparison of quantity discount schemes. We focus on the distribution network example presented in Lal and Staelin (1984) that has been extensively studied by Wang (2001, 2002). The distribution network concerns a large US manufacturer (supplier) with 1128 dealers (buyers). The buyers are clustered into seven groups, and the relevant data of each group are presented in Table 1, where $G_i$ is the number of buyers in group $i$. The example of Lal and Staelin (1984) includes a fixed processing cost (denoted by $H_{i,k}$ in Table 1) for every order the supplier receives from buyer $i$. Although the models we have presented so far have not explicitly included this additional cost, they can be readily modified (by changing the definition of $H_{i,k}$) to operate under this more general cost structure. In the subsequent analysis of the Lal and Staelin (1984) distribution network, we determine the discount policies with the modified versions of our models. The objective function of the modified (DSDP: $T_S$) incorporates the number of buyers in a group as the weight of the total savings achieved by a single buyer in that group.

In order to compute the supply chain coordination efficiency of various discount schemes, we need to first determine the total supply chain cost when buyers choose their initial order quantities (or replenishment periods) independently. The supplier determines his optimal replenishment policy to meet the buyers’ demand with these initial order quantities (Table 2). As discussed in Section 2.1, the supplier needs to solve a lot sizing problem where the planning horizon is determined by the Least Common Multiple (LCM) of buyers’ initial replenishment periods. We assume that there are 100 unit time periods per year in the discretized problem (i.e., $\tau = 100$). We then compute the initial replenishment periods of buyer groups as 26, 18, 14, 11, 9, 12, and 11 unit time periods for groups 1 through 7, respectively. Since LCM of integers 26, 18, 14, 11, 9, 12, and 11 is equal to 36,036, the supplier solves a lot sizing problem where the planning horizon is equal to 36,036 unit time periods (or $36,036 \times 100 / 360 = 360$ years) to determine his optimal replenishment policy. Under his optimal replenishment

<table>
<thead>
<tr>
<th>Group $i$</th>
<th>$D_i$</th>
<th>$v_i \times r$</th>
<th>$S_i$</th>
<th>$\mathcal{J}_i$</th>
<th>$G_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>362</td>
<td>11.50</td>
<td>145</td>
<td>81</td>
<td>632</td>
</tr>
<tr>
<td>2</td>
<td>1658</td>
<td>10.00</td>
<td>283</td>
<td>147</td>
<td>363</td>
</tr>
<tr>
<td>3</td>
<td>4191</td>
<td>10.50</td>
<td>407</td>
<td>305</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>9228</td>
<td>10.00</td>
<td>526</td>
<td>445</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>14,966</td>
<td>9.50</td>
<td>634</td>
<td>573</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>18,565</td>
<td>9.25</td>
<td>1305</td>
<td>1275</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>25,346</td>
<td>9.00</td>
<td>1305</td>
<td>1275</td>
<td>6</td>
</tr>
<tr>
<td>Supplier</td>
<td>1,809,777</td>
<td>3.00</td>
<td>4000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
policy, the supplier’s annual cost is equal to 1,049,432.97 (a spreadsheet file detailing this solution is available from the authors). We note that this approximately quantifies the best the supplier could do without coordination, and is used as a benchmark while we compute coordination benefits. A crude approach that uses a more practical planning horizon, for instance by choosing the replenishment periods among from even numbers, is possible in general. Because we will be analyzing coordination benefits closely, we did not incorporate such an approximation here.

In Table 3 we present a comparison of five different discount schemes. First four discount schemes are based on the models presented in this paper:

1. Optimal integer ratio policy when anticipated net savings of buyers are equal to zero and the RP Act is not in effect.
2. Optimal integer ratio policy when anticipated net savings of buyers are equal to 50% of sub-chains’ anticipated net savings and the RP Act is not in effect.
3. Optimal integer ratio policy when anticipated net savings of buyers are equal to zero and the RP Act is in effect.
4. Optimal integer ratio policy when anticipated net savings of buyers are equal to 50% of sub-chains’ anticipated net savings and the RP Act is in effect.

The last discount scheme is taken from Wang (2001):

Table 3
Comparison of discount schemes

<table>
<thead>
<tr>
<th>Group</th>
<th>Discount scheme</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Qi</td>
<td>DC_j</td>
<td>Qi</td>
<td>DC_j</td>
<td>Qi</td>
<td>DC_j</td>
</tr>
<tr>
<td>1</td>
<td>115.84</td>
<td>0.0561</td>
<td>115.84</td>
<td>0.0717</td>
<td>101.36</td>
<td>0.0499</td>
</tr>
<tr>
<td>2</td>
<td>397.92</td>
<td>0.0929</td>
<td>397.92</td>
<td>0.0929</td>
<td>348.18</td>
<td>0.0344</td>
</tr>
<tr>
<td>3</td>
<td>670.56</td>
<td>0.0182</td>
<td>670.56</td>
<td>0.0235</td>
<td>586.74</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>1476.48</td>
<td>0.0880</td>
<td>1476.48</td>
<td>0.0880</td>
<td>1291.92</td>
<td>0.0734</td>
</tr>
<tr>
<td>5</td>
<td>2394.56</td>
<td>0.1265</td>
<td>2394.56</td>
<td>0.1265</td>
<td>2095.24</td>
<td>0.1428</td>
</tr>
<tr>
<td>6</td>
<td>2970.40</td>
<td>0.0530</td>
<td>2970.40</td>
<td>0.0565</td>
<td>2599.10</td>
<td>0.1599</td>
</tr>
<tr>
<td>7</td>
<td>4055.36</td>
<td>0.0787</td>
<td>4055.36</td>
<td>0.0787</td>
<td>3548.44</td>
<td>0.1946</td>
</tr>
</tbody>
</table>

Net savings

| Buyers  | 5,620.94 | 68,415.17 | 96,104.76 | 112,519.17 | 137,876.00 |
| Supplier| 132,255.06| 16,414.41 | 43,651.95 |
| Supply chain| 137,876.00 | 112,519.17 | 62,004.37 |

Efficiencies

| Buyers  | 4.1%   | 49.6%  | 69.7%  |
| Supplier| 95.9%  | 32.0%  | 11.9%  | 31.7%  |
| Supply chain| 100.0% | 81.6%  | 81.6%  | 45.0%  |

– Sharing the next lower break point.
The first discount scheme ignores the RP Act and guarantees perfect coordination in the supply chain, under the discrete time line assumption, when only integer-ratio policies are allowed. The third discount scheme operates with the objective of maximizing supplier’s net savings under the RP Act. In this regard, the discount scheme is similar to the discount scheme of Wang (2001). The power-of-two coordination strategy presented in Wang (2001) (the fifth scheme in Table 3) generates a net savings of 43,651.95 (or 31.7% of the total savings) for the supplier whereas with the third discount scheme, the supplier receives a net savings of 44,104.00 (or 32% of the total savings). The third discount scheme finds a solution with a higher supply chain coordination efficiency (81.6% vs 45.0% in the fifth discount scheme), and, because it relies only on the quantity discounts to achieve time coordination and compliance with the RP Act, ends up transferring more savings to the buyers (68,415.17 vs 18,352.41 in the fifth discount scheme). The discrepancy in supplier’s net savings in favor of scheme 3 is somewhat a counter intuitive finding because of the heuristic nature of our approach. Moreover, although it may achieve higher coordination benefits, a solution obtained using our scheme 3 is not typically expected to generate more savings for the supplier because it relies only on quantity discounts to achieve time coordination, and is most likely paying a price for something scheme 5 achieves at no additional cost due to the implicit requirement that buyers should comply with the supplier’s base planning period. In this particular example, the discrepancy is partly due to the rounding errors, and partly to the fact that there are multiple buyers in every group, and both our method and Wang (2001) are able to handle the big problem only in an approximate manner.

To compare the time-coordination aspects of discount Schemes 3 and 5, a closer look at buyer group seven will be helpful. According to scheme 5, the supplier reveals his base-planning period as $T_0 = 0.1501$, and requires buyers to comply with it in choosing their own order quantities (see the numerical example provided in Section 4 of Wang, 2001). For a buyer who is in group seven the initial order quantity is computed as 2711.15 when the time line is continuous. The initial order of this buyer group is above the third break-point of the proposed 3-break-point discount schedule, therefore, because the cheapest price is feasible with the initial order quantity, the optimal strategy of a buyer in group seven is to stay with the initial order quantity. However, since 2711.15 divided by the annual demand of a buyer in group seven does not fulfill the power-of-two multiple or ratio requirement, i.e., $2711.15/25,346 = 0.1070 < 0.1501/2^k$ or $0.1070 < 0.1501/2^k$, $k \in \{0, 1, 2, \ldots\}$, the buyers in group seven choose $2^{0} = 1$ as the optimal power-of-two multiple, and their new order quantity is computed as $0.1501 \times 25,346 \times 1 = 3804.43$. Without this requirement, the discount scheme cannot force buyers in the seventh group to change their initial order quantities. Therefore, the discount schemes presented in Wang (2001) (or Wang, 2004) are not in full compliance with the RP Act, because some buyers are forced to deviate from their optimal order quantities under the discount scheme.

The second discount scheme considers the case where buyers form their expectations based on their partial view of the supply chain without the RP Act. Even when buyers’ expectations are incorporated into the discount scheme design process, the supplier is able to fully coordinate the supply chain and gain more than 95% of coordination savings. When sub-chains are analyzed in isolation, we observe that the setup cost (holding cost) of the supplier is much higher (lower) than the buyers’ setup (holding) costs. This is a typical case where coordination does not generate too much savings. Let us take the second buyer group with $Q_0^2 = 298.44$. When the supplier proposes a new order quantity of $Q_2^{IR} = 397.32$, a buyer in Group 2 would compute the anticipated net savings as

| Table 4 | Overall results |
|-----------------|-----------------|-----------------|-----------------|
|               | Supply chain efficiency | Supplier’s efficiency | Buyer’s efficiency |
|               | ANS setting | 1 | 2 | 3 | ANS setting | 1 | 2 | 3 | ANS setting | 1 | 2 | 3 |
| With the RP Act (%) | 94.7 | 93.2 | 94.6 | 83.8 | 70.0 | 64.6 | 10.9 | 23.1 | 30.0 |
| Without the RP Act (%) | 100.0 | 98.2 | 99.8 | 100.0 | 88.9 | 82.1 | 0.0 | 9.3 | 17.7 |
| Difference (%) | −5.3 | −5.1 | −5.2 | −16.2 | −18.9 | −17.5 | 10.9 | 13.8 | 12.3 |
ANS_2 = \max \left\{ 0, \frac{1}{2} N_{\text{sc}}(397.92) \right\},

where \( N_{\text{sc}}(397.92) = T_{\text{c}}(298.44, n_2(298.44)) + T_{\text{c}}(298.44) - T_{\text{c}}^2(397.92, n_2(397.92)) - T_{\text{c}}^2(397.92) \). \( n_2(298.44) \) and \( n_2(397.92) \) are computed as 7 and 5, respectively, and ANS_2 is computed as \( \max(0, -10.104) = 0 \). Therefore, the supplier’s discount scheme covers only the additional cost of a buyer in the second group. The additional cost is equal to \( T_{\text{c}}(397.92) - T_{\text{c}}(298.44) = 154.0844 \), and the total discount offered by the supplier is also equal to \( 1658 \cdot 0.092934 / C_0(154.0844) \).

In this problem instance, the buyers’ share of coordination benefits increases dramatically when the RP Act is in effect. Even when buyers’ expectations are not incorporated into the discount scheme design process, i.e., in the third discount scheme, buyers are able to receive 49.6% of the maximum coordination savings. Their share of savings increases to 69.7% when their expectations are part of the discount scheme design process and the RP Act is in effect (the fourth discount scheme).

3. Numerical analysis

The primary objective of the numerical experiments we conduct is to analyze the effect of vertical information sharing on the overall efficiency of the supply chain and the allocation of this efficiency between the supplier and the buyers. Therefore, we will solve a given instance of the supply chain coordination problem under three settings of buyers’ net savings expectations: 1) Buyers’ anticipated net savings are zero, i.e., they expect to be compensated for their additional costs due to coordination (note that this corresponds to the traditional Stackelberg modeling of the supply chain coordination problem with no vertical information sharing); 2) Buy-
ers’ anticipated net savings are given by Eq. (1), which computes the net savings expectation of a buyer as half of the gains associated with the isolated supply sub-chain; 3) Buyers’ anticipated net savings are given by Eq. (2), which models the case where buyers have access to the supplier’s total demand information and adjust their expectations accordingly.

As a secondary objective, we would like to observe the impact of the non-price discriminatory discount scheme requirement, i.e., the RP Act, on the efficiency of the supply chain and its allocation between the channel members. Although some negative effect can be anticipated due to the non-price discriminatory discount scheme requirement, as argued in Viscusi et al. (2000), we are more interested in studying the magnitude of this efficiency loss. Therefore, a given test problem will be solved under six different scenarios that combine the three settings of the buyers’ net savings expectations with the absence or presence of the non-price discriminatory discount scheme requirement.

In generating the random test instances, we treat some problem parameters as experiment factors, as we describe in the next subsection, and later analyze the impact of these factors on the supply chain efficiency and its allocation.

### 3.1. Test problems

In order to define an instance of the single-supplier/multiple buyers supply chain problem, we need to determine the values of the following parameters: \( S_c \) and \( v_i \): supplier’s cost parameters; \( N \): number of buyers; \( S_i \) and \( v_i \): buyer \( i \)’s cost parameters, \( i = 1, 2, \ldots, N \); \( D_i \): buyer \( i \)’s demand, \( i = 1, 2, \ldots, N \); \( \tau \): number of time periods per year; \( r \): holding cost rate. In generating the test problems, these parameters are selected as follows:
where
\[ q \in \{50, 10\}, \{100, 10\}, \{250, 10\} \}, \]
\[ N \in \{2, 3, 4, 5\}, \]
\[ (S, v_i) = (S_{21},q_{1,1}, v_S,q_{2,2}), \]
and
\[ (a, b) \in \{(0.95, 1.05), (0.85, 1.15), (0.75, 1.25), (0.65, 1.35)\}, \quad i = 1, 2, \ldots, N, \]
\[ D_i = 10,000 \frac{\rho_i}{1000}, \]
where
\[ \rho_i \sim N(\mu, \sigma^2) \quad \text{and} \quad (\mu, \sigma^2) \in \{(1, 0.05), (1, 0.10), (1, 0.20), (1, 0.30)\}, \quad i = 1, 2, \ldots, N, \]
\[ \tau = 100, \]
\[ r = 20\%. \]

Except for the interest rate parameter \( r \), the number of base periods parameter \( \tau \), and the variable cost of the product for the supplier \( v_S \), the parameters listed above are treated as experiment factors. Our preliminary runs have indicated the results are not very sensitive to the choice of the \( \tau \) value, therefore \( \tau \) is chosen as 100 unit time periods/year, corresponding to a maximum of two order placement opportunities per week. Since we generate values for the variable cost of the product for buyer \( i \), i.e., \( v_i \), in relation to the value of \( v_S \), and treat their ratio as an experiment factor, \( v_S \) is kept at a constant value. The setup cost of the supplier is treated at three different levels. The setup cost of each buyer is defined in relation to the setup cost of the supplier at five different multiplier levels \( (x_1) \) and four different levels of variability \( (\rho_{i,1}) \). Similarly, the variable cost of buyer \( i \) is defined in relation to the variable cost of the product for the supplier at four different multiplier levels \( (x_2) \) and four different levels of variability \( (\rho_{i,2}) \). The demand faced by each buyer is also an experiment factor that is induced by a normal distribution with four different variance values. In general, the experiment factors lead to a total of \( 3 \times 4 \times 5 \times 4 \times 4 = 3840 \) problem instances.

We use the script language of OPL (2002) as the programming environment, and the MIP models are solved using the MIP solver of CPLEX (2002). When we consider the CPU times for determining the supply chain costs of the centralized solution, the differences can be expressed as
\[ D = TC_{SC}(RP) + \sum_{i=1}^{N} TC_i(Q_i) - TC_{LB}(T_{SC}^{RP}), \]
where \( T_{SC}^{RP} \) denotes supplier’s optimal replenishment interval according to Roundy (1985), and \( TC_{LB}(T_{SC}^{RP}) \) is the total cost of the centralized system. \( \Delta \) is an upper bound on the maximum coordination savings possible. We then compute the difference between our procedure’s total supply chain cost when the RP Act is not in effect and buyers’ expectations are all zero (Scenario 1) and again Roundy (1985)’s integer ratio lower bound, expressed as
\[ \delta = TC_{SC}(RP: T_{SC}^{RP}) + \sum_{i=1}^{N} TC_i(Q_i) - TC_{LB}(T_{SC}^{RP}), \]
where \( T_{SC}^{RP} \) is the supplier’s optimal replenishment interval obtained using our procedure when the anticipated net savings of buyers are all zero and the RP Act is not in effect. Scenario 1 compensates the buyers for their additional costs in the coordinated solution without any
restriction on the design of the discount scheme, and therefore the objective function of the supplier becomes equivalent to maximizing the net savings of the supply chain generating the overall best for the supply chain, and allocating all of the net savings to himself. \(\delta\) expressed as a percentage of \(\Delta\) (i.e., \(\frac{\Delta}{\Delta} \times 100\)) gives the portion of the maximum savings our approach is able to capture, and has an average value of 89% across all problems. This can be seen roughly as an indicator of the approximation quality of our modeling approach that operates over the discretized time line.

Next, for some test problems we observe that the supplier’s net savings may not be strictly positive for all six scenarios. Due to the non-price discriminatory discount scheme requirement, the net savings of the supplier given by \(\text{TC}_S(RP: T_S^*) - \text{TC}_S(RP^0)\) where \(\text{TC}_S(RP^0)\) denotes the pre-coordination lot sizing solution, may be negative. In such a case, the supplier chooses not to implement the discount scheme. In other words, as the supplier becomes more pessimistic about the expectations of the buyers, he is more likely not to find coordination beneficial. In 18% of the test problems, coordination is not found to be beneficial under at least one scenario. We observe that with Scenario 1, our approach leads to a coordination policy with positive returns for the supplier for more than 99% of the cases. Since our procedure ensures time coordination using only quantity discounts without enforcing additional restrictions on the buyers, our cost for a coordinated supply chain may be high especially under a unified pricing assumption. From here on, these test problems are excluded from our analysis and the results are reported for the 82% of cases for which all scenarios lead to positive coordination benefits for the supplier.

In Table 4, we report the average values of supply chain, supplier’s, and buyers’ efficiencies. These efficiencies are computed as the coordination benefits of the respective scenario normalized with respect to Scenario 1. The benefit associated with Scenario \(j\) is given by \(B_{SC} = \text{TC}_S(RP) + \sum_{i=1}^{N} \text{TC}_i(Q_i^*) - (\text{TC}_S(RP : T_S^*) + \sum_{i=1}^{N} \text{DC}_i^*(D_i))\) where \(\text{TC}_S(RP : T_S^*)\) denotes the supplier’s setup and holding costs under Scenario \(j\) with a replenishment interval of \(T_S^*\). \(\text{TC}_S(RP)\) denotes the total cost of buyer \(i\) using her new optimal order quantity \(BP_i\) and \(\text{DC}_i^*(D_i)\) is the per unit discount paid to buyer \(i\) for an order quantity of \(BP_i\). The supply chain efficiency of Scenario \(j\) is given by \(\frac{B_{SC}}{B_{SC}^1} \times 100\). Supplier’s efficiency corresponds to the portion collected by the supplier, given by \(\frac{\text{TC}_S(RP) - (\text{TC}_S(RP : T_S^*) + \sum_{i=1}^{N} \text{DC}_i^*(D_i))}{\sum_{i=1}^{N} \text{TC}_i(Q_i^*) - (\sum_{i=1}^{N} \text{TC}_i(BP_i) - \sum_{i=1}^{N} \text{DC}_i^*(D_i))} \times 100\), and buyers’ efficiency is the portion collected by the buyers, given by \(\frac{\sum_{i=1}^{N} \text{TC}_i(Q_i^*) - (\sum_{i=1}^{N} \text{TC}_i(BP_i) - \sum_{i=1}^{N} \text{DC}_i^*(D_i))}{\sum_{i=1}^{N} \text{TC}_i(Q_i^*) - (\sum_{i=1}^{N} \text{TC}_i(BP_i) - \sum_{i=1}^{N} \text{DC}_i^*(D_i))} \times 100\).

The impact of the RP Act on the efficiency of the supply chain turns out to be small in our experiments, and we observe that this impact is less than 5.3%. This implies that the Act does not create significant efficiency losses, and from a supply chain coordination perspective, the additional cost of designing a non-price discriminatory discount scheme seems to be reasonable. The Act seems to change the allocation of the net savings in favor of buyers. The change is more pronounced with the second ANS setting.

The impact of incorporating buyers’ anticipated net savings on supply chain efficiency is rather minimal. However, when the RP Act is in effect, significant changes are observed in the allocation of the net savings. When the first setting of anticipated net savings is used, all of the net savings allocated to the buyers are due to the presence of the RP Act, and are not large in magnitude (\(\frac{99.9}{94.7} = 11.5\%\)). When the second (third) setting is used to model buyers’ expectations, the net savings transferred to buyers’ amount to \(\frac{23.1}{97.2} = 24.8\%\) (\(\frac{30.0}{94.6} = 31.7\%\)) of the net savings generated by the coordinated solution. This is in line with the fact that the buyers possess incrementally more information in settings two and three, and therefore are able to obtain increasingly larger portions of the net savings. In the second setting where each buyer anticipates to share the net savings equally with the supplier based on her view of the supply sub-chain, the share of the actual savings buyers obtain is still relatively small at 23.1% of supply chain efficiency. In other words, the supplier is able to generate a hidden margin for himself when dealing with multiple buyers who operate without horizontal information sharing. The supplier transfers 23.1% of supply chain efficiency to buyers, and loses 100 – 93.2 = 6.8% of supply chain efficiency in coordinating the supply chain. Hence, the magnitude of the hidden margin is equal to 50% – 23.1% – (100% – 93.2%) = 20.0%, and this seems to be decreasing with the number of buyers in the supply chain as we will discuss when we analyze the results presented in Tables 5 and 6. Although the additional information of total demand does actually reduce the magnitude of the supplier’s hidden margin, it still remains considerably large at 50% – 30.0% – (100% – 94.6%) = 14.6%. On
buyers’ behalf, the supplier’s hidden margin can be interpreted as the cost of buyers’ not sharing information horizontally.

In Tables 5 and 6 we report the averages over the treatment levels of our experimental factors. In general, we observe that high levels of supply chain efficiency can be attained regardless of the parameter settings. When the RP Act is not in action, Table 5 shows that allocation of the efficiencies is not greatly affected by the variation in the treatment levels. The highest variation is observed as a response to the variation in the $\xi_1$ parameter which gives the $\frac{S}{S_i}$ ratio. In Table 6, we report the averages when the RP Act is in effect, and observe the same pattern with parameter $\xi_1$. Moreover, we now observe that the allocation between the supplier and buyers is more sensitive to the number of buyers ($N$), and the variation in the demand levels of individual buyers as captured by the $(a,b)$ range. As the number of buyers increases, or the variability in buyers’ demand levels increases, buyers’ total share of the supply chain coordination benefits increases. In a sense, a larger supply chain with higher demand variability might be harder to coordinate from the supplier’s perspective when the RP Act forces him not to employ price discrimination.

4. Concluding remarks

In this paper we have addressed the coordination problem in a single-supplier/multiple-buyer supply chain with vertical information sharing. Our modeling constitutes a first attempt at incorporating buyers’ expectations into the supply chain coordination problem. We have modeled each buyer’s net savings expectations based on her limited view of the entire supply chain which consists of herself and the supplier only, and then incorporated these expectations into the modeling of the supply chain conducted by the supplier, which results in a constrained Stackelberg game. We have considered both price discriminatory and non-price discriminatory approaches. The quantity discount schemes presented in this paper achieve time coordination without any additional requirement for buyers to comply with.

Our main observation is the fact that, when dealing with multiple buyers in a vertical information sharing setting, the supplier is able to generate a hidden margin which affects the allocation of the net savings in his favor. The RP Act does not seem to change the hidden margin of the supplier to a large extent. This may not be surprising, as the RP Act was initiated in an attempt to maintain a competitive environment among buyers. Our numerical analysis has shown that the buyers’ share of the net savings increases as more information becomes available to them, but remains far below the 50% share they believe they would achieve. The lack of horizontal information sharing, which can be attributed to the competition among buyers, benefits the supplier by providing him with the hidden margin. The analysis of the optimal strategy of the buyers as to the horizontal sharing of information requires further studies that look at the cost of loss of competitiveness due to horizontal information sharing relative to the magnitude of the hidden margin of the supplier under vertical information sharing.

Alternative modeling constructs to capture buyers’ expectations remain a possibility. A parametric analysis of the buyer’s expectations might be revealing. Finally, modeling supply chain structures with partial information and/or that rely on repetitive games is another possible future direction.

Appendix A. An MIP formulation of supplier’s optimal replenishment policy

The length of the renewal period can be simply determined by finding the least common multiple of integers $k_i^0$, $i = 1, 2, \ldots, N$. Let $T$ be the length of the renewal period, in terms of number of unit time periods. We now define the following notation for the optimal replenishment policy problem of the supplier:

- $R_t$: required shipment quantity in period $t$, $t = 1, 2, \ldots, T$,
- $H$: supplier’s cost for holding one unit in the inventory during the unit time interval $t$,
- $I_t$: supplier’s inventory level at the end of period $t$,
- $P_t$: supplier’s replenishment quantity at the beginning of period $t$,
- $S_t$: supplier’s 0–1 setup decision at the beginning of period $t$. 
The parameter \( R_t \) can be computed as follows:
\[
R_t = \sum_{i:\mod(t,\pi)+1=0} k_i D_i \frac{D_i}{\tau}. 
\]
Similarly, the parameter \( H \) can be determined as follows: \( H = v_s r \frac{t}{\tau} \). Given the above parameters and decision variables, the optimal replenishment can be formulated as follows:
\[
\text{(RP)} \quad \min \sum_{t=1}^{T} (S_S S_t + H I_t) 
\]
s.t.
\[
\begin{align*}
I_1 - P_1 + R_1 &= 0, \\
I_t - I_{t-1} - P_t + R_t &= 0, \quad t = 2, 3, \ldots, T, \\
MS_t - P_t &\geq 0, \quad t = 1, 2, \ldots, T, \\
I_{t-1} - R_t + MS_t &\geq 0, \quad t = 2, 3, \ldots, T, \\
I_{t-1} - R_t - R_{t+1} + MS_t + MS_{t+1} &\geq 0, \quad t = 2, 3, \ldots, T - 1, \\
S_t &\in \{0, 1\}, \quad t = 1, 2, \ldots, T,
\end{align*}
\]
where \( M \) is a sufficiently large positive constant. In (RP), constraint sets (10) and (11) are the inventory balance requirements, constraint set (12) forces the system to incur the setup cost when the replenishment quantity of a period is greater than zero. Constraint sets (13) and (14), which are similar to those introduced in Kalymon (1972), are redundant constraints introduced in an attempt to solve the problem more efficiently. The problem (RP) can be considerably simplified by eliminating decision variables associated with periods where required shipment quantities are equal to zero. Once these decision variables are eliminated, the period index can be re-defined as the shipment index, and the holding cost parameters can be re-computed using the time differences between successive shipment points in the time line.

**Appendix B. Supply chain total cost functions with modified demand**

In the continuous case, which is an approximation of the discrete time-line version of the problem, when the initial order quantity of the buyer is \( Q_i^0 \), the total cost of the sub-chain that consists of buyer \( i \) and the supplier is equal to
\[
\frac{D_i}{Q_i^0} \left( \frac{S_S}{n_i} + S_B \right) + \frac{Q_i^0}{2} (n_i v_S + (v_i - v_S)) r,
\]
where \( n_i \) is the lot-size multiplier of the supplier. When buyer \( i \) becomes the only customer of the supplier with a demand equal to \( \mathcal{D} \), the initial order quantity that the supplier faces becomes \( \sqrt{D \cdot Q_i^0} \), and, when the lot-size multiplier of the supplier is equal to \( n_i \), the total cost of the supply chain is given by
\[
\frac{\mathcal{D}}{\sqrt{D \cdot Q_i^0}} \left( \frac{S_S}{n_i} + S_B \right) + \frac{\sqrt{D \cdot Q_i^0}}{2} (n_i v_S + (v_i - v_S)) r,
\]
or
\[
\sqrt{\frac{\mathcal{D}}{D_i}} \left( \frac{D_i}{Q_i^0} \left( \frac{S_S}{n_i} + S_B \right) + \frac{Q_i^0}{2} (n_i v_S + (v_i - v_S)) r \right),
\]
which indicates that, for lot size multiplier \( n_i \), the total cost of the sub-chain increases by a factor of \( \sqrt{D_i} \).

When the sub-chain that consists of buyer \( i \) and the supplier is optimized jointly for a lot-size multiplier \( n_i \), the total cost of the sub chain is given by \( \sqrt{2 D_i (S_i + \frac{S_S}{n_i}) (n_i v_S + (v_i - v_S)) r} \). Similarly, when the demand of
buyer $i$ is changed as $D$, and the lot-size multiplier is set equal to $n_i$, the total cost increases by a factor of $\sqrt{\frac{2}{D}}$, and becomes

$$\sqrt{2(D)} \left( S_i + \frac{S_S}{n_i} \right) (n_i v_S + (v_i - v_S)) r.$$ 

In both cases, the optimal lot size multiplier of the supplier remains the same, and the coordination savings of the supply chain increases by a factor of $\sqrt{\frac{D}{n_i}}$.

References


CPLEX. ILOG Cplex 8.0. ILOG Inc.; 2002.


