SOLUTIONS to HW 5

3.52 a. The possible outcomes for this problem are:

(Yes, ≥50), (Yes, ≥50), (No, ≥50), (No, ≥50)

b. We can find reasonable probabilities the 4 sample points by dividing each frequency by the total sample size of 358. The estimates of the probabilities are:

<table>
<thead>
<tr>
<th>Permit Drug at Home</th>
<th>Less than 50</th>
<th>50 or more</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>.475</td>
<td>.363</td>
<td>.838</td>
</tr>
<tr>
<td>No</td>
<td>.134</td>
<td>.228</td>
<td>.162</td>
</tr>
<tr>
<td>Totals</td>
<td>.609</td>
<td>.591</td>
<td>1.000</td>
</tr>
</tbody>
</table>

c. Define the following event:

A: (Provider permits home use of abortion drug)
B: (Provider has case load of less than 50 abortions)

\[ P(A) = .838 \]

d. \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = .838 + .609 - .475 = .972 \]

e. \[ P(A \cap B) = .475 \]

d. The event B is made up of the following sample points:

\( (FI, F), (AU, F), (UF, F), (K, F), (AP, F), (OG, F), (FP, F), (O, F), (U, F) \)

Then,


\[ = 63/981 + 60/981 + 18/981 + 9/981 + 9/981 + 3/981 + 9/981 + 0.981 + 3/981 + 9/981 \]

\[ = 1.74/981 = .177 \]

e. The event C is made up of the following sample points: \( (AU, F) \) and \( (AU, N) \)

Then,

\[ P(C) = P(AU, F) + P(AU, N) = 60/981 + 178/981 = .238 \]

f. The event D is made up of the following sample point: \( AU, F \)

Then,

\[ P(D) = P(AU, F) = 60/981 = .061 \]

g. The event E is made up of the following sample point: \( FI, N \)

Then,

\[ P(E) = P(FI, N) = 53/981 = .054 \]

h. \[ P(F | FI) = 63/116 = .543 \]

i. \[ P(AU | N) = 178/807 = .221 \]

j. \[ P(U | F) = 9/174 = .052 \]

k. The event "not fire or fireplace" would include \( AU, FU, K, AP, OG, O, \) and \( U \).

\[ P(AU \cup FU \cup K \cup AP \cup OG \cup O \cup U | N) \]

\[ = (178 + 345 + 18 + 63 + 73 + 19 + 42)/807 = .738/807 = .914 \]
3.160  a. Let S denote the event that an insect travels toward the pheromone and F the event that an insect travels toward the control. The sample space associated with the experiment of releasing five insects has 32 simple events:

SSSSS  FSSFS  SSFFS  FFSFS
SSSSF  FSSFF  SSFFF  FFFFF
SSSFS  SFSSF  SSFFT  FFFFS
SSFSS  SFSSF  SSFFF  FFFSF
SSFSS  SFSSF  FFSSF  FFFF
SFSFS  SFSFF  FFFFS  FFFFF
FSSSS  SSSSF  FSFFS  FFFFS
FSSSF  SSSSF  FSSFS  FSSSF

If the pheromone under study has no effect, then each simple event is equally likely and occurs with probability \( \frac{1}{32} = 0.03125 \).

b. The probability that all five insects travel toward the pheromone is \( P(SSSSS) = \frac{1}{32} = 0.03125 \).

c. The probability that exactly four of the five insects travel toward the pheromone is:
\[
P(SSSSF) + P(SSFFS) + P(SSFFF) + P(FSSF) + P(FSSS) = \frac{5}{32} = 0.15625
\]

d. Since the probability that exactly four of the five insects travel toward the pheromone is not small (0.15625), this is not an unusual event. (This probability was computed assuming the pheromone has no effect.)

Note that you can solve this question using combinations! We will learn binomial probabilities in Chp. 4. You may give it a try now!

4.6 The values \( x \) can assume are 1, 2, 3, 4, or 5. Thus, \( x \) is a discrete random variable.

4.8 Annual rainfall can take on any value in an interval. Thus, annual rainfall is a continuous random variable. The number of ant species can assume only a countable number of outcomes. Thus, number of ant species is a discrete random variable.

4.16  a. This is a valid distribution because \( \sum p(x) = 1 \) and \( p(x) \geq 0 \) for all values of \( x \).

b. This is not a valid distribution because \( \sum p(x) = 0.95 \neq 1 \).

c. This is not a valid distribution because one of the probabilities is negative.

d. The sum of the probabilities over all possible values of the random variable is greater than 1, so this is not a valid probability distribution.

4.18  a. \( P(x \leq 0) = P(x = -2) + P(x = -1) + P(x = 0) = 0.10 + 0.15 + 0.40 = 0.65 \)

b. \( P(x > -1) = P(x = 0) + P(x = 1) + P(x = 2) = 0.40 + 0.30 + 0.05 = 0.75 \)

c. \( P(-1 \leq x \leq 1) = P(x = -1) + P(x = 0) + P(x = 1) = 0.15 + 0.40 + 0.30 = 0.85 \)

d. \( P(x < 2) = 1 - P(x = 2) = 1 - 0.05 = 0.95 \)

e. \( P(-1 < x < 2) = P(x = 0) + P(x = 1) = 0.40 + 0.30 = 0.70 \)

f. \( P(x < 1) = P(x = -2) + P(x = -1) + P(x = 0) = 0.10 + 0.15 + 0.40 = 0.65 \)
4.22  a. To find probabilities, change the percents given in the table to proportions by dividing by 100. The probability distribution for x is:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>.40</td>
<td>.54</td>
<td>.02</td>
<td>.04</td>
</tr>
</tbody>
</table>

b. \( P(x \geq 3) = P(x = 3) + P(x = 4) = .02 + .04 = .06 \).

4.24  a. \( p(0) = \frac{\binom{20}{0} \left( \frac{100 - 20}{3 - 0} \right)}{100 \choose 3} = \frac{20! \cdot 80!}{0! \cdot 20! \cdot 3! \cdot 77!} = \frac{1}{\frac{80 \cdot 79 \cdot 78 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 97 \cdot 96 \cdot 95 \cdot \ldots \cdot 1}} \cdot \frac{1}{\frac{100 \cdot 99 \cdot 98 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 97 \cdot 96 \cdot 95 \cdot \ldots \cdot 1}} = \frac{82,160}{161,700} = .508 \)

b. \( p(1) = \frac{\binom{20}{1} \left( \frac{100 - 20}{3 - 1} \right)}{100 \choose 3} = \frac{20! \cdot 80!}{1! \cdot 19! \cdot 2! \cdot 78!} = \frac{1}{\frac{20 \cdot 19 \cdot 18 \cdot \ldots \cdot 1}{2 \cdot 1 \cdot 78 \cdot 77 \cdot 76 \cdot \ldots \cdot 1}} \cdot \frac{1}{\frac{100 \cdot 99 \cdot 98 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 97 \cdot 96 \cdot 95 \cdot \ldots \cdot 1}} = \frac{63,200}{161,700} = .391 \)

c. \( p(2) = \frac{\binom{20}{2} \left( \frac{100 - 20}{3 - 2} \right)}{100 \choose 3} = \frac{20! \cdot 80!}{2! \cdot 18! \cdot 1! \cdot 79!} = \frac{1}{\frac{20 \cdot 19 \cdot 18 \cdot \ldots \cdot 1}{2 \cdot 1 \cdot 18 \cdot 17 \cdot 16 \cdot \ldots \cdot 1}} \cdot \frac{1}{\frac{100 \cdot 99 \cdot 98 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 97 \cdot 96 \cdot 95 \cdot \ldots \cdot 1}} = \frac{15,200}{161,700} = .094 \)

d. \( p(3) = \frac{\binom{20}{3} \left( \frac{100 - 20}{3 - 3} \right)}{100 \choose 3} = \frac{20! \cdot 80!}{3! \cdot 17! \cdot 0! \cdot 80!} = \frac{1}{\frac{20 \cdot 19 \cdot 18 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 17 \cdot 16 \cdot 15 \cdot \ldots \cdot 1}} \cdot \frac{1}{\frac{100 \cdot 99 \cdot 98 \cdot \ldots \cdot 1}{3 \cdot 2 \cdot 1 \cdot 97 \cdot 96 \cdot 95 \cdot \ldots \cdot 1}} = \frac{1,140}{161,700} = .007 \)

4.28  a. \( P(x \geq 10) = p(10) + p(15) + p(30) + p(50) + p(99) = .05 + .085 + .01 + .004 + .001 = .150 \)

b. \( P(x < 0) = p(-4) + p(-2) + p(-1) = .02 + .06 + .07 = .15 \)
4.30 Suppose we define the following events:

\(A\): \{Child has an attached earlobe\}
\(N\): \{Child does not have an attached earlobe\}

From the graph, \(P(A) = 1/4 = .25\). Thus, \(P(N) = 1 - P(A) = 1 - .25 = .75\)

If seven children are selected, there will be \(2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128\) possible outcomes for the 7 children. Of these outcomes, there is one that has all A's, 7 that have 6 A's and 1 N, 21 that have 5 A's and 2 N's, 35 ways to get 4 A's and 3 N's, 35 ways to get 3 A's and 4 N's, 21 ways to get 2 A's and 5 B's, 7 ways to get 1 A and 6 B's, and 1 way to get 0 A's and 7 N's. The list of the outcomes is:

\[
\begin{align*}
AAAAAAA & \quad NAAAAA & \quad AANNAA & \quad NNNNAA & \quad NAANNN & \quad NNANNN \\
AAAAAAN & \quad AANNAA & \quad ANANAA & \quad NNAAAN & \quad NANAAN & \quad NANNAN \\
AAAAANA & \quad AANANA & \quad ANANNA & \quad NANAAN & \quad ANNNAN & \quad ANNNNN
\end{align*}
\]

and so on …

\(P(AAAAAAA) = .25^7 = .000061035 = .0001 = P(x = 7)\)

\(P(AAAAAAN) = (.25)^6(.75) = .000183105.\) Since there are 7 ways to select 6 A's and 1 N, \(P(x = 6) = 7(.000183105) = .0013\)

\(P(AAAAANN) = (.25)^5(.75)^2 = .000549316.\) Since there are 21 ways to select 5 A's and 2 N's, \(P(x = 5) = 21(.000549316) = .0115\)

\(P(AAAAANN) = (.25)^4(.75)^3 = .001647949.\) Since there are 35 ways to select 4 A's and 3 N's, \(P(x = 4) = 35(.001647979) = .0577\)

\(P(AAANNNN) = (.25)^3(.75)^4 = .004943847.\) Since there are 35 ways to select 3 A's and 4 N's, \(P(x = 3) = 35(.004943847) = .1730\)

\(P(AAANNNN) = (.25)^2(.75)^5 = .014831542.\) Since there are 21 ways to select 2 A's and 5 N's, \(P(x = 2) = 21(.014831542) = .3115\)

\(P(ANNNNNN) = (.25)(.75)^6 = .044494628.\) Since there are 7 ways to select 1 A's and 6 N's, \(P(x = 1) = 7(.044494628) = .3115\)

\(P(NNNNNNN) = (.75)^7 = .133483886.\) Since there is 1 way to select 0 A's and 7 N's, \(P(x = 0) = .1335\)