case is that the consumer will choose a "boundary solution" where she consumes only one good. At this point, her indifference curve will not be tangent to her budget line.

When a consumer has kinks in her indifference curves, she may choose a bundle that is located at a kink. In the problems with kinks, you will be able to solve for the demand functions quite easily by looking at diagrams and doing a little algebra. Typically, instead of finding a tangency equation, you will find an equation that tells you "where the kinks are." With this equation and the budget equation, you can then solve for demand.

You might wonder why we pay so much attention to kinky indifference curves, straight line indifference curves, and other "funny cases." Our reason is this. In the funny cases, computations are usually pretty easy. But often you may have to draw a graph and think about what you are doing. That is what we want you to do. Think and fiddle with graphs. Don't just memorize formulas. Formulas you will forget, but the habit of thinking will stick with you.

When you have finished this workout, we hope that you will be able to do the following:

- Find demand functions for consumers with Cobb-Douglas and other similar utility functions.
- Find demand functions for consumers with quasilinear utility functions.
- Find demand functions for consumers with kinked indifference curves and for consumers with straight-line indifference curves.
- Recognize complements and substitutes from looking at a demand curve.
- Recognize normal goods, inferior goods, luxuries, and necessities from looking at information about demand.
- Calculate the equation of an inverse demand curve, given a simple demand equation.

6.1 (0) Charlie is back—still consuming apples and bananas. His utility function is \( U(x_A, x_B) = x_A x_B \). We want to find his demand function for apples, \( x_A(p_A, p_B, m) \), and his demand function for bananas, \( x_B(p_A, p_B, m) \).

(a) When the prices are \( p_A \) and \( p_B \) and Charlie's income is \( m \), the equation for Charlie's budget line is \( p_A x_A + p_B x_B = m \). The slope of Charlie's indifference curve at the bundle \((x_A, x_B)\) is \(-MU_1(x_A, x_B)/MU_2(x_A, x_B) = \) _____________. The slope of Charlie's budget line is _____________. Charlie's indifference curve will be tangent to his budget line at the point \((x_A, x_B)\) if the following equation is satisfied: ________________.
(b) You now have two equations, the budget equation and the tangency equation, that must be satisfied by the bundle demanded. Solve these two equations for $x_A$ and $x_B$. Charlie's demand function for apples is $x_A(p_A, p_B, m) =$ ______, and his demand function for bananas is $x_B(p_A, p_B, m) =$ ______.

(c) In general, the demand for both commodities will depend on the price of both commodities and on income. But for Charlie's utility function, the demand function for apples depends only on income and the price of apples. Similarly, the demand for bananas depends only on income and the price of bananas. Charlie always spends the same fraction of his income on bananas. What fraction is this? __________________________.

6.2 (0) Douglas Cornfield's preferences are represented by the utility function $u(x_1, x_2) = x_1^2 x_2^3$. The prices of $x_1$ and $x_2$ are $p_1$ and $p_2$.

(a) The slope of Cornfield's indifference curve at the point $(x_1, x_2)$ is __________________________.

(b) If Cornfield's budget line is tangent to his indifference curve at $(x_1, x_2)$, then $\frac{p_1 x_1}{p_2 x_2} =$ ______ (Hint: Look at the equation that equates the slope of his indifference curve with the slope of his budget line.) When he is consuming the best bundle he can afford, what fraction of his income does Douglas spend on $x_1$? __________________________.

(c) Other members of Doug's family have similar utility functions, but the exponents may be different, or their utilities may be multiplied by a positive constant. If a family member has a utility function $U(x_1, x_2) = a x_1^b x_2^c$ where $a$, $b$, and $c$ are positive numbers, what fraction of his or her income will that family member spend on $x_1$? __________________________.

6.3 (0) Our thoughts return to Ambrose and his nuts and berries. Ambrose's utility function is $U(x_1, x_2) = 4\sqrt{x_1} + x_2$, where $x_1$ is his consumption of nuts and $x_2$ is his consumption of berries.

(a) Let us find his demand function for nuts. The slope of Ambrose's indifference curve at $(x_1, x_2)$ is ______. Setting this slope equal to the slope of the budget line, you can solve for $x_1$ without even using the budget equation. The solution is $x_1 =$ __________________________.
(b) Let us find his demand for berries. Now we need the budget equation. In Part (a), you solved for the amount of \( x_1 \) that he will demand. The budget equation tells us that \( p_1x_1 + p_2x_2 = M \). Plug the solution that you found for \( x_1 \) into the budget equation and solve for \( x_2 \) as a function of income and prices. The answer is \( x_2 = \)__________.

(c) When we visited Ambrose in Chapter 5, we looked at a "boundary solution," where Ambrose consumed only nuts and no berries. In that example, \( p_1 = 1 \), \( p_2 = 2 \), and \( M = 9 \). If you plug these numbers into the formulas we found in Parts (a) and (b), you find \( x_1 = \)_______, and \( x_2 = \)_______ . Since we get a negative solution for \( x_2 \), it must be that the budget line \( x_1 + 2x_2 = 9 \) is not tangent to an indifference curve when \( x_2 \geq 0 \). The best that Ambrose can do with this budget is to spend all of his income on nuts. Looking at the formulas, we see that at the prices \( p_1 = 1 \) and \( p_2 = 2 \), Ambrose will demand a positive amount of both goods if and only if \( M > \)__________.

6.4 (0) Donald Fribble is a stamp collector. The only things other than stamps that Fribble consumes are Hostess Twinkies. It turns out that Fribble’s preferences are represented by the utility function \( u(s, t) = s + \ln t \) where \( s \) is the number of stamps he collects and \( t \) is the number of Twinkies he consumes. The price of stamps is \( p_s \) and the price of Twinkies is \( p_t \). Donald’s income is \( m \).

(a) Write an expression that says that the ratio of Fribble’s marginal utility for Twinkies to his marginal utility for stamps is equal to the ratio of the price of Twinkies to the price of stamps. _________ (Hint: The derivative of \( \ln t \) with respect to \( t \) is \( 1/t \), and the derivative of \( s \) with respect to \( s \) is 1.)

(b) You can use the equation you found in the last part to show that if he buys both goods, Donald’s demand function for Twinkies depends only on the price ratio and not on his income. Donald’s demand function for Twinkies is__________.

(c) Notice that for this special utility function, if Fribble buys both goods, then the total amount of money that he spends on Twinkies has the peculiar property that it depends on only one of the three variables \( m \), \( p_t \), and \( p_s \), namely the variable _______. (Hint: The amount of money that he spends on Twinkies is \( p_t(p_s, p_t, m) \).)
8-ounce cans

40
30
20
10
0
10
20
30
40
16-ounce cans

(a) At these prices, which size can will she buy, or will she buy some of each?

(b) Suppose that the price of 16-ounce beers remains $1 and the price of 8-ounce beers falls to $.55. Will she buy more 8-ounce beers?

(c) What if the price of 8-ounce beers falls to $.40? How many 8-ounce beers will she buy then?

(d) If the price of 16-ounce beers is $1 each and if Shirley chooses some 8-ounce beers and some 16-ounce beers, what must be the price of 8-ounce beers?

(e) Now let us try to describe Shirley’s demand function for 16-ounce beers as a function of general prices and income. Let the prices of 8-ounce and 16-ounce beers be $p_8$ and $p_{16}$, and let her income be $m$. If $p_{16} < 2p_8$, then the number of 16-ounce beers she will demand is _______. If $p_{16} > 2p_8$, then the number of 16-ounce beers she will demand is _______. If $p_{16} =$ _______ $p_8$, she will be indifferent between any affordable combinations.

6.6 (0) Miss Muffet always likes to have things “just so.” In fact the only way she will consume her curds and whey is in the ratio of 2 units of whey per unit of curds. She has an income of $20. Whey costs $.75 per unit. Curds cost $1 per unit. On the graph below, draw Miss Muffet’s budget line, and plot some of her indifference curves. (Hint: Have you noticed something kinky about Miss Muffet?)
(a) How many units of curds will Miss Muffet demand in this situation?  

How many units of whey?  

Whey  

(b) Write down Miss Muffet's demand function for whey as a function of the prices of curds and whey and of her income, where \( p_c \) is the price of curds, \( p_w \) is the price of whey, and \( m \) is her income. \( D(p_c, p_w, m) = \)  

(Hint: You can solve for her demands by solving two equations in two unknowns. One equation tells you that she consumes twice as much whey as curds. The second equation is her budget equation.)

6.7 (1) Mary's utility function is \( U(b, c) = b + 100c - c^2 \), where \( b \) is the number of silver bells in her garden and \( c \) is the number of cockle shells. She has 500 square feet in her garden to allocate between silver bells and cockle shells. Silver bells each take up 1 square foot and cockle shells each take up 4 square feet. She gets both kinds of seeds for free.

(a) To maximize her utility, given the size of her garden, Mary should plant _____ silver bells and _____ cockle shells. (Hint: Write down her "budget constraint" for space. Solve the problem as if it were an ordinary demand problem.)

(b) If she suddenly acquires an extra 100 square feet for her garden, how much should she increase her planting of silver bells?  

_____ How much should she increase her planting of cockle shells?
the area of this trapezoid by applying the formula

\[
\text{Area of a trapezoid} = \text{base} \times \frac{1}{2} (\text{height}_1 + \text{height}_2).
\]

In this case we have \( A = 5 \times \frac{1}{2} (100 + 50) = 375 \).

Example: Suppose now that the consumer is purchasing the 5 units at a price of $50 per unit. If you require him to reduce his purchases to zero, how much money would be necessary to compensate him?

In this case, we saw above that his gross benefits decline by $375. On the other hand, he has to spend \( 5 \times 50 = 250 \) less. The decline in net surplus is therefore $125.

Example: Suppose that a consumer has a utility function \( u(x_1, x_2) = x_1 + x_2 \). Initially the consumer faces prices \((1, 2)\) and has income 10. If the prices change to \((4, 2)\), calculate the compensating and equivalent variations.

Answer: Since the two goods are perfect substitutes, the consumer will initially consume the bundle \((10, 0)\) and get a utility of 10. After the prices change, she will consume the bundle \((0, 5)\) and get a utility of 5. After the price change she would need $20 to get a utility of 10; therefore the compensating variation is \( 20 - 10 = 10 \). Before the price change, she would need an income of 5 to get a utility of 5. Therefore the equivalent variation is \( 10 - 5 = 5 \).

14.1 (0) Sir Plus consumes mead, and his demand function for tankards of mead is given by \( D(p) = 100 - p \), where \( p \) is the price of mead in shillings.

\( (a) \) If the price of mead is 50 shillings per tankard, how many tankards of mead will he consume?

\( (b) \) How much gross consumer’s surplus does he get from this consumption?

\( (c) \) How much money does he spend on mead?

\( (d) \) What is his net consumer’s surplus from mead consumption?

14.2 (0) Here is the table of reservation prices for apartments taken from Chapter 1:

\[
\begin{array}{cccccccccc}
\text{Person} & A & B & C & D & E & F & G & H \\
\text{Price}  & 40 & 25 & 30 & 35 & 10 & 18 & 15 & 5 \\
\end{array}
\]
Calculus 15.4 (0) The demand for kitty litter, in pounds, is \( \ln D(p) = 1,000 - p + \ln m \), where \( p \) is the price of kitty litter and \( m \) is income.

(a) What is the price elasticity of demand for kitty litter when \( p = 2 \) and \( m = 500 \)? ________ When \( p = 3 \) and \( m = 500 \)? ________ When \( p = 4 \) and \( m = 1,500 \)?

(b) What is the income elasticity of demand for kitty litter when \( p = 2 \) and \( m = 500 \)? ________ When \( p = 2 \) and \( m = 1,000 \)? ________ When \( p = 3 \) and \( m = 1,500 \)?

(c) What is the price elasticity of demand when price is \( p \) and income is \( m \)? ________ The income elasticity of demand?

Calculus 15.5 (0) The demand function for drangles is \( q(p) = (p + 1)^{-2} \).

(a) What is the price elasticity of demand at price \( p \)?

(b) At what price is the price elasticity of demand for drangles equal to \(-1\)?

(c) Write an expression for total revenue from the sale of drangles as a function of their price. ________ Use calculus to find the revenue-maximizing price. Don’t forget to check the second-order condition.

(d) Suppose that the demand function for drangles takes the more general form \( q(p) = (p + a)^{-b} \) where \( a > 0 \) and \( b > 1 \). Calculate an expression for the price elasticity of demand at price \( p \). ________ At what price is the price elasticity of demand equal to \(-1\)?

15.6 (0) Ken’s utility function is \( u_K(x_1, x_2) = x_1 + x_2 \) and Barbie’s utility function is \( u_B(x_1, x_2) = (x_1 + 1)(x_2 + 1) \). A person can buy 1 unit of good 1 or 0 units of good 1. It is impossible for anybody to buy fractional units or to buy more than 1 unit. Either person can buy any quantity of good 2 that he or she can afford at a price of $1 per unit.