23.4 (0) Consider a competitive industry with a large number of firms, all of which have identical cost functions \( c(y) = y^2 + 1 \) for \( y > 0 \) and \( c(0) = 0 \). Suppose that initially the demand curve for this industry is given by \( D(p) = 52 - p \). (The output of a firm does not have to be an integer number, but the number of firms does have to be an integer.)

(a) What is the supply curve of an individual firm? \( S(p) = p/2 \). If there are \( n \) firms in the industry, what will be the industry supply curve?
\[
Y = np/2.
\]

(b) What is the smallest price at which the product can be sold? \( p^* = 2 \).

(c) What will be the equilibrium number of firms in the industry? (Hint: Take a guess at what the industry price will be and see if it works.)

\textbf{Guess at} \( p^* = 2 \). This gives \( D(p) = 52 - 2 = n2/2 \), which says \( n^* = 50 \).

(d) What will be the equilibrium price? \( p^* = 2 \). What will be the equilibrium output of each firm? \( y^* = 1 \).

(e) What will be the equilibrium output of the industry? \( Y^* = 50 \).
(f) Now suppose that the demand curve shifts to \( D(p) = 52.5 - p \). What will be the equilibrium number of firms? (Hint: Can a new firm enter the market and make nonnegative profits?) If a new firm entered, there would be 51 firms. The supply-demand equation would be \( 52.5 - p = 51p/2 \). Solve for \( p^* = 105/53 < 2 \). A new firm would lose money. Therefore in equilibrium there would be 50 firms.

(g) What will be the equilibrium price? Solve \( 52.5 - p = 50p/2 \) to get \( p^* = 2.02 \). What will be the equilibrium output of each firm? \( y^* = 1.01 \). What will be the equilibrium profits of each firm? Around \( 0.02 \).

(h) Now suppose that the demand curve shifts to \( D(p) = 53 - p \). What will be the equilibrium number of firms? 51. What will be the equilibrium price? 2.

(i) What will be the equilibrium output of each firm? \( y = 1 \). What will be the equilibrium profits of each firm? Zero.
22.1 (0) Remember Otto’s brother Dent Carr, who is in the auto repair business? Dent found that the total cost of repairing $s$ cars is $c(s) = 2s^2 + 100$.

(a) This implies that Dent’s average cost is equal to $2s + 100/s$, his average variable cost is equal to $2s$, and his marginal cost is equal to $4s$. On the graph below, plot the above curves, and also plot Dent’s supply curve.
(b) If the market price is $20, how many cars will Dent be willing to repair? 5. If the market price is $40, how many cars will Dent repair? 10.

(c) Suppose the market price is $40 and Dent maximizes his profits. On the above graph, shade in and label the following areas: total costs, total revenue, and total profits.

22.2 (0) A competitive firm has the following short-run cost function: 
\[ c(y) = y^3 - 8y^2 + 30y + 5. \]

(a) The firm’s marginal cost function is \( MC(y) = 3y^2 - 16y + 30. \)

(b) The firm’s average variable cost function is \( AVC(y) = y^2 - 8y + 30. \) (Hint: Notice that total variable costs equal \( c(y) - c(0). \))

(c) On the axes below, sketch a graph of the marginal cost function and the average variable cost function.

(d) Average variable cost is falling as output rises if output is less than 4 and rising as output rises if output is greater than 4.

(e) Marginal cost equals average variable cost when output is 4.
(f) The firm will supply zero output if the price is less than 14.

(g) The smallest positive amount that the firm will ever supply at any price is 4. At what price would the firm supply exactly 6 units of output? 42.