Instructions

- Stick your name tag or write your name in the space above.
- There are 5 questions. Answer all of them in the pages provided to you.
- Show all your work, and explain. The quality of the explanation determines the grade. Just writing a number without explanations will not earn you any credit.
- Write clearly. We can’t give you partial credit if we don’t understand what you are doing.
- Exam is closed book and notes. No cellphones or calculators allowed. Good luck!
- Time allowed: 2 hours
Question 1.
Suppose that I consider goods $x_1$ and $x_2$ as perfect substitutes (by this I mean "real" perfect substitutes, that is, one $x_1$ is perfectly substitutable for some $x_2$). The prices of goods are $p_1 = 2$ and $p_2 = 3$. My income is 60 dollars ($m = 60$).

1. Write a utility function that represents these preferences.

2. Draw at least two indifference curves for these preferences.

3. What will the slope of the IC’s be?

4. Write the equation of and draw the budget line. On your graph, put numbers for the intercepts, label the slopes.

5. Show on your graph the consumption bundle that I will choose. What will my consumption of $x_1$ and $x_2$ be?

Show your work. Explain your answer.
Question 2.
Suppose that $x_1$ is an inferior good, and suppose that its price increases. Write and explain the directions of the substitution and income effects of the price increase on the amount of $x_1$ consumed (that is, would SE lead to an increase/decrease in $x_1$ consumption? Would IE lead to an increase/decrease in $x_1$ consumption?). Is it possible that the consumer will end up consuming more $x_1$ after the price increase? When would this be the case? (explain by making reference to SE and IE) What is the name given to such goods (for which demand varies positively with price)?

Show your work. Explain your answer.
**Question 3.**

John has the following utility function \( U(x_1, x_2) = \ln x_1 + \ln x_2 \). The prices of the goods are given by \( p_1, p_2 \) and income is given by \( m \). (Hint: the derivative of \( U \) with respect to \( x_1 \) is equal to \( \frac{1}{x_1} \). The derivative of \( U \) with respect to \( x_2 \) is equal to \( \frac{1}{x_2} \)).

1. Does the utility function \( V(x_1, x_2) = x_1 + x_1x_2 \) represent the same preferences as John’s?


3. What is the value of John’s MRS at the consumption bundle (2,4)? What does this mean in words?

4. If the price ratio \( (p_1/p_2) \) is 1, could (2,4) be an optimal bundle for John? Why/why not?

5. Derive John’s demand functions for good 1 and good 2.

6. Now assume that \( m = 100 \) and \( p_1=1, p_2=1 \). What is the optimal bundle (of course, you can use the demand functions you found in the previous part, you don’t have to solve it again)?

7. What is John’s utility level at the optimal bundle?

8. Now assume that \( p_1 \) increases to 2. Find the new optimal bundle (again, you can use the demand function). Plot the old optimal bundle, old budget line, old IC, new optimal bundle, new budget line, new IC on a graph.

**Show your work. Explain your answer.**
Question 3. Additional Answer Space
Question 4.
The demand function for good 1 is given by \( x_1(p_1, p_2, m) = \frac{2m}{2p_1 + p_2} \).

1. Is \( x_1 \) a normal good?

2. Are good 1 and good 2 substitutes or complements?

Show your work. Explain your answer. Just answering with one word will not earn you credit, even if it is correct.
**Question 5:** Suppose that demand for broccoli is given by $Q^d = 400 - 2P^d$, and supply is given by $Q^s = 3P^s$

1. What is the price elasticity of demand when $P=100$? (Hint: Your answer should be a number) Show how you calculated it.

2. Find the equilibrium quantity and price, and plot it on a graph. Label *axes, intercepts, slopes* of the demand and supply functions.

3. Find the consumer surplus in this equilibrium. Find the producer surplus in this equilibrium.

4. Now suppose that the government gives the producers a per-unit subsidy in the amount of 20 dollars (this means that for each unit they sell, they get $20 from the government, in addition to what they are getting from the consumers). Find the new equilibrium price and quantity and prices, that is:
   
   (a) What is the price received by the suppliers?
   (b) What is the price paid by the demanders?
   (c) What is the new equilibrium quantity?

5. Illustrate the effect of the subsidy, and the effective prices faced by demanders and suppliers clearly on a graph, as well as the amount of the subsidy.
Question 5—Additional Answer Space