1) a) \( U(x_1, x_2) = x_1 + x_2 \)

(Notice that there could be other \( U \) functions that could represent these preferences: e.g. \( V(x_1, x_2) = 2x_1 + 2x_2 \).)

b) 

![Graph](image)

c) \( MRS = \text{Slope of IC} = -1 \). This is because the person would be willing to give up at most 1 unit of \( x_2 \) to get one more \( x_1 \).

d) \( p_1 x_1 + p_2 x_2 = m \)

\[ 2x_1 + 3x_2 = 60 \]

![Graph](image)

e) Since \( |MRS| > \frac{p_1}{p_2} \), maximize \( \sum \) of income.

Optimal consumption will occur where all income is spent on \( x_1 \). That is, \( (x_1^*, x_2^*) = \left( \frac{m}{p_1}, 0 \right) \)

\[ = (30, 0) \]
$p_1 \uparrow \Rightarrow SE$ leads to an increase in $X_1$ consumption because $X_1$ is now relatively more expensive.

When $p_1 \uparrow$, purchasing power declines. Since $X_1$ is an inferior good, when purchasing power $\downarrow$, IE leads to an increase in $X_1$ consumption.

The total effect of the price change depends on which effect dominates.

If $SE > IE \Rightarrow p_1 \uparrow \Rightarrow X_1 \downarrow$

If $IE > SE \Rightarrow p_1 \uparrow \Rightarrow X_1 \uparrow$.

So, it is possible that the consumer consumes more $X_1$ after the price increase. This happens when $IE > SE$. Such goods are called Giffen goods.
\[ U(x_1, x_2) = \ln x_1 + \ln x_2. \]

a) Check if MRS's are equal:

\[ \text{MRS}_v = -\frac{\mu_1}{\mu_2} = \left( 1 + \frac{x_2}{x_1} \right) \]

\[ \text{MRS}_u = -\frac{x_2}{x_1} \]

\[ \text{MRS}_v \neq \text{MRS}_u \Rightarrow \text{The functions do not represent the same preferences.} \]

b) \[ |\text{MRS}| = \frac{x_2}{x_1} \]

As \[ |\text{MRS}| \] declines as \( x_1 \) increases, IC's may be convex. That is, \( \frac{\partial (|\text{MRS}|)}{\partial x_1} = -\frac{x_2}{x_1^2} < 0. \)

\[ c) \text{MRS} = \frac{x_2}{x_1}, \text{ so } \text{MRS} = -2 \text{ at } (x_1, x_2) = (2, 4). \]

This means that when he has 2 units of good 1 and 4 units of good 2, John is willing to give up at most 2 units of \( x_2 \) to get 1 more \( x_1 \).

d) For optimality, we need \( |\text{MRS}| = \frac{\bar{p}_1}{\bar{p}_2} \). This means that since \( \frac{\bar{p}_1}{\bar{p}_2} = 1 \), \( x_2 = x_1 \) if \( (x_1, x_2) \) is an optimal bundle. Since \( 4 \neq 2 \), (2, 4) cannot be optimal at this price ratio. In fact, we have \( |\text{MRS}| > \frac{\bar{p}_1}{\bar{p}_2} \) here.

e) From the tangency condition, \[ \text{MRS} = -\frac{\bar{p}_1}{\bar{p}_2}, \Rightarrow \frac{x_2}{x_1} = \frac{\bar{p}_1}{\bar{p}_2} \]

\[ x_1 \bar{p}_1 = x_2 \bar{p}_2. \quad (1) \]

We also know that the optimal bundle should satisfy the budget constraint. Therefore, \( \bar{p}_1 x_1 + \bar{p}_2 x_2 = m \) \( (2) \).
Plugging (1) into (2), we get:

\[ p_1x_1 + p_2x_2 = m \Rightarrow x_1^* = \frac{m}{2p_1}, \quad x_2^* = \frac{m}{2p_2} \]

f) \( m = 100, \quad p_1 = 1, \quad p_2 = 1 \)

\[ x_1^* = \frac{100}{2} = 50 \]

\[ x_2^* = \frac{100}{2} = 50 \]

g) \( u(50, 50) = \ln(50) + \ln(50) = 2 \ln(50) \)

h) \( p_1 = 2, \quad p_2 = 1 \Rightarrow \quad x_1^* = \frac{100}{2(2)} = 25 \]

\[ x_2^* = \frac{100}{2(1)} = 50. \]
\[ x_1(p_1, p_2, m) = \frac{2m}{2p_1 + p_2} \]

1) \(x_1\) is a normal good because \(\frac{\partial x}{\partial m} = \frac{2}{2p_1 + p_2} > 0\). That is, as \(m\) \(\uparrow\), \(x_1\) \(\uparrow\).

2) Goods 1 and Goods 2 are complements because 
\[ \frac{\partial x(p_1, p_2, m)}{\partial p_2} = -2m (2p_1 + p_2)^{-2} < 0. \] That is, as \(p_2\) \(\uparrow\), \(x_1\) decreases.

\[
\begin{align*}
Q &= 400 - 2p_d^d \\
Q &= 3p_s^s \\
\frac{dQ}{dp} &= \frac{\partial Q}{\partial p} \\
\frac{dQ}{dp} &= -2,100 \\
\frac{dQ}{dp} &= -1
\end{align*}
\]

\[ Q = 240 \]

\[ P = 200 \]

\[ S = 80 \]

\[ P = p_d^d + 20 \]

\[ Q = 3(88) = 264 \]

\[ CD = \frac{240, 120}{2} = 240, 60 \]

\[ PS = \frac{240, 80}{2} = 240, 40 \]

\[ Q = 3(p_d^d + 20) \]

\[ 400 - 2p_d^d = 3(p_d^d + 20) \]

\[ 400 - 2p_d^d = 3p_d^d + 60 \]

\[ 3p_d^d = 340 \Rightarrow p_d^d = 68 \]

\[ p_s^s = 88 \]
5- (Continued)

\[
\begin{align*}
P &= 88 \\
P^* &= 80 \\
Q &= 68 \\
S &= 20
\end{align*}
\]