Summer 2007, EXAM 1

Instructions: There are seven questions in this exam. Please inspect the exam and make sure you have all 7 questions. You may only use your calculator and one A4 size page of notes. Do all your work on the paper provided. Please use one sheet per question. If you use the back of a page, make sure to indicate that. You may not exchange any kind of material with another student.

Remember: You must show your work to get proper credit.

Academic Honesty Code: Koç University Academic Honesty Code stipulates that “copying from others or providing answers or information, written or oral, to others is cheating.” By taking this exam, you are assuming full responsibility for observing the Academic Honesty Code.

NAME: KEY LECTURE TIME:

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**Question 1:**
A bad form of cancer occurs in 3% of all Aleuts. A screening test has been developed that is good, but not perfect. The test will label 5% of healthy Aleuts as having cancer, and will label 2% of Aleuts with cancer as not having cancer. What percentage of the people who are labeled as having cancer by the test, actually do have cancer?

**Question 2:**
Two dice are thrown. One is red, and one is white. The following events are defined:
- E = 'Red die is 5'
- F = 'Total is 7'
- G = 'Total is 10'

a) Show that \( P(F|E) = P(F) \)
b) Show that \( P(G|E) \neq P(G) \)
c) If you are told that the red die shows 5, do you want to bet on whether the dice show a total of 7, or 10? Why?

**Question 3:**
a) How many ways can you put 7 pictures in 5 frames?
b) How many ways can you put 3 pictures in 7 frames?

Note: In (a) and (b), each frame is in a separate room.
c) Two out of seven pictures are fake. If you choose four pictures at random, what is the probability that you have only one fake picture among the four?

**Question 4:**
The temperature for the next 6 days is going to be 32, 29, 28, 31, 34, and 37. (All numbers are in degrees centigrade).
a) Find the average and variance of the temperatures for the next 6 days
b) Find the average and variance of the temperatures for the next 6 days in degrees Fahrenheit. ([Fahrenheit] = 32 + 1.8[Celsius]. Example: [86F] = 32 + 1.8[30C]).

**Question 5:**
\[ y = 0.56 + 0.6x \]
is the least square line (regression line) for the following pairs of measurements: \( x_1, 1.6 \), \( x_2, 2.4 \), \( x_3, 3.0 \), \( x_4, 3.4 \), \( x_5, 4.0 \)
a) What is \( \bar{x} \), the average of the x-values?
b) What is \( S_{yy} \)?
c) Assuming that \( x_1 = 1, x_2 = 1.5, x_3 = 2, x_4 = 2.5, x_5 = 3 \), find the correlation coefficient.
d) What is the sum of squares of vertical errors for the regression line?

**Question 6:**
A, B, and C are three events defined on a sample space. \( P(A) = 0.36 \), \( P(B) = 0.5 \),
\( P(C) = 0.50 \), \( P(A \cap B) = 0.18 \), \( P(A \cap C) = 0.11 \), \( P(B \cap C) = 0.20 \), and \( P(A \cap B \cap C) = 0.08 \),
a) Investigate the independence of A and B using the product of their probabilities.
b) Investigate the independence of A and C using \( P(C|A) \).
c) Investigate the independence of B and C using \( P(B|C) \).
d) Can you find the sum of the probabilities of all simple events outside \( A \cup B \cup C \) ?

**Question 7:**
The probability that Selma’s boyfriend will come to the party tonight is only 0.7, because he has to study for a statistics test. If he doesn’t come, then the probability that Selma will leave the party with somebody else is 0.1. If he comes, she will of course leave the party with her boyfriend. What is the probability that she leaves the party by herself? Given that she didn’t leave the party by herself, what is the probability that she left with her boyfriend?
Q1

\[ P(C/\neg C) = \frac{P(C \land \neg C)}{P(C)} \]

\[ = \frac{(0.03)(0.98)}{(0.03)(0.98) + (0.97)(0.02)} \]

\[ = 37.74\% \]
Q2

a.) \[ P(F/E) = \frac{P(F \cap E)}{P(E)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6} \]

\[ P(F) = P\{1,6, (6,1), (2,5), (5,2), (4,3), (3,3)\} = \frac{1}{6} \]

b.) \[ P(G \cap E) = \frac{1}{36} \quad P(G) = P\{(5,5), (4,4), (3,3)\} = \frac{3}{36} \]

\[ \Rightarrow P(G/E) = \frac{1}{3}, \text{ but } P(G) = \frac{1}{12} \]

c.) Bet on "dice show total of 10", because the prob. has improved.

Q3

a.) \[ \frac{P(7,5)}{2!} = \frac{7!}{2!} = 2520 \]

b.) \[ P(7,3) = \frac{7!}{9!} = 210 \]

c.) \[ \frac{\binom{2,1}{7,4} \binom{5,3}}{\binom{7,4}} = \frac{2 \cdot \frac{5!}{3!2!}}{\frac{7!}{4!3!}} = \frac{20}{35} = 0.5714 \]
Q4
a. ) \( \bar{x} = \frac{31.83}{6} = 5.3 \)

b. ) \( \bar{y} = 32 + 1.8 \cdot \bar{x} = 88.3 \)

Q5
a. ) \( \bar{y} - \beta_1 \bar{x} = 60.6 \)
\( 2.88 - 1.16 \bar{x} = 0.56 \Rightarrow \bar{x} = 2 \)

OR \( \bar{y} - \beta_1 \bar{x} = 0.56 \)
\( 2.88 - 1.6 \bar{x} = 0.56 \Rightarrow \bar{x} = 1.45 \)

b. ) \( S_{y|y} = \frac{\sum y_i^2}{n} = \frac{1}{n} (\sum y_i)^2 = 44.88 - \frac{1}{5} (14.4)^2 = 5.408 \)

\( t = \frac{2.9}{\sqrt{5.408 / 2.5}} = 0.9935 \)
\[ S_{xy} = 31.7 - \frac{1}{5}(10)(14.5) = 2.9 \]
\[ S_{xx} = 22.5 - \frac{1}{6}(10)^2 = 2.5 \]
\[ \beta_1 = \sqrt{1.16} \quad \checkmark \]
\[ \beta_0 = 2.88 - 1.16(2) = 0.56 \quad \checkmark \]

\[
\begin{array}{c|c|c|c|c|c|c}
\times & 1 & 1.5 & 2 & 2.5 & 3 \\
\hline
y = 0.56 + 1.16x & 1.72 & 2.3 & 2.88 & 3.46 & 4.04 \\
\end{array}
\]

\[ \Rightarrow \text{Sum of squares } E_1^2 + E_2^2 + \ldots + E_5^2 \]
\[ = (0.12)^2 + (0.1)^2 + (0.12)^2 + (0.06)^2 + (0.04)^2 \]
\[ = 0.0544 \]
9. \(P(A) = 0.36 \quad P(B) = 0.5 \rightarrow P(A \cap B) = 0.18\)

**INDEPENDENT**

\[ b. \quad P(C/A) = \frac{P(C \cap A)}{P(A)} = \frac{0.11}{0.36} = \frac{0.3056}{\text{DEP}} \]

\[ \neq P(C) = 0.5 \quad \text{DEPENDENT} \]

c. \(P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{0.2}{0.5} = 0.4\)

\[ \neq P(B) = 0.5 \]

d. \(P(A \cup B \cup C) = 1 - \left[ P(A) + P(B) + P(C) - P(A \cap C) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C) \right] \]

\[ = 1 - \left[ 0.36 + 0.5 + 0.5 - 0.18 - 0.11 - 0.20 + 0.08 \right] = \frac{0.05}{\text{DEP}} \]
\[ P(\text{with her/boyfriend}) = \frac{P(A \cap B)}{P(B)} = \frac{0.7}{0.073} = 0.959 \]\n\[ P(\text{by herself}) = 0.27 \]